[This question paper contains 4 printed pages.]

3046 *Your Roll No.* ....

MEC **J**

Paper – CE.501

(Advanced Mathematics and Numerical Techniques)

Time : 3 hours *Maximum Marks : 100*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer any **five** *questions.*
Each question carries equal mark.

1. (a) Apply Gauss-Seidel method to solve the system of following equations

$$10x - 2y - z - w = 3$$
$$-2x + 10y - z - w = 15$$
$$-x - y + 10z - 2w = 27$$
$$-x - y - 2z + 10w = -9$$

upto the end of fourth iteration and correct to 5th places of decimals.

(b) Find the Fourier series of the function defined in $(-2, 2)$ as follows :

$$f(x) = 2, \quad -2 \le x < 0$$
$$= x, \quad 0 < x \le 2.$$

(10,10)

2. (a) Solve the following system of equations

$$x + 2y + z = 4$$

$$2x - 3y - z = -3$$

$$3x + y + 2z = 3$$

by Crout's method.

(b) Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \leq x \leq \pi$. (10,10)

3. (a) Find the components of a vector in polar coordinate system whose components in Cartesian coordinate system are $\dot{x}, \dot{y}$ and $\ddot{x}, \ddot{y}$.

(b) Show that the contraction of the outer product of the tensors A^i and B_j is an invariant. (10,10)

4. (a) Show that $e^{\frac{1}{2}x\left(t-\frac{1}{t}\right)}$ is the generating function of the Bessel's functions.

(b) Prove that

$$\frac{d}{dx}\left[J_n^2(x) + J_{n+1}^2(x)\right] = 2\left[\frac{n}{x}J_n^2(x) - \frac{n+1}{x}J_{n+1}^2(x)\right]$$

(10,10)

5. (a) Show that

$$\int_{-1}^{1} x\, P_n(x)\; P_{n-1}(x)\, dx = \frac{2n}{4n^2 - 1}$$

(b) Using Rodrigue's formula, find the polynomial expression for $P_4(x)$ and $P_5(x)$. (10,10)

6. (a) Solve

$$(x^2 - y^2 - z^2)p + 2xyq = 2xz$$

(b) A slider in a machine moves along a fixed straight rod. Its distance x cm along the rod is given below for various valves of the time t sec. Find the velocity and acceleration of the slider when t = 0.3 sec.

t :	0	0.1	0.2	0.3	0.4	0.5	0.6
x :	30.13	31.62	32.87	33.64	33.95	33.81	33.24

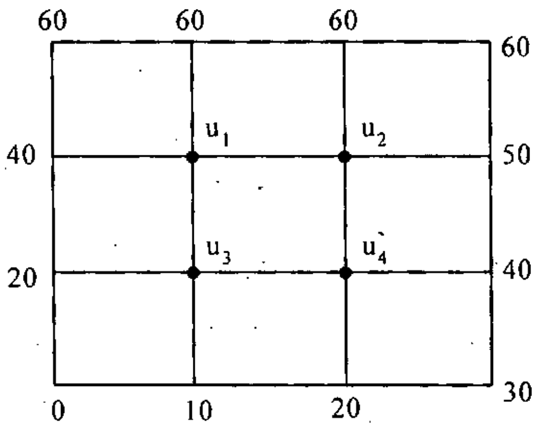
(10,10)

7. (a) Given $\frac{dy}{dx} = x^2(1 + y)$, $y(1) = 1$,

$y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$.

Evaluate y(1.4) by Milne's Predictor-Corrector method.

(b) Using the given boundary valves, solve Laplace equation $\nabla^2 u = 0$ at the nodal points of the square grid shown in figure up to 3rd places of decimal.



(10,10)