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Your Roll No. ....

3433 **A**

M. Tech/II Sem.

NUCLEAR SCIENCE & TECHNOLOGY

Paper NST–608 : Mathematical & Numerical Methods in Nuclear Engineering

Time : 3 Hours *Maximum Marks : 70*

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt **all** questions.*

1. Attempt any ***two*** of the following :

   (a) Find the minimum number of iterations needed to calculate the root of a function in the interval [0, 1] to an accuracy of 10^{-6} using the bisection method. 2.5

   (b) Find the interval $[a, b]$ which encloses a unique fixed point of the function $g(x) = \sin x$. 2.5

   (c) Derive Newton–Raphson formula for approximating the root of a function $f(x)$. 2.5

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2. Attempt any ***two*** of the following. Any method of root finding can be used.

(a) Solve $x = \cos x$ to an accuracy of 10^{-5}. 5

(b) The energy distribution of neutrons for a neutron source is given by 5

$$n(E) = \sqrt{E}\ \exp(-E/1.3).$$

Find the energy E, correct upto 3 places of decimal, where the flux $n(E)$ is maximum.

(c) The probability distribution function for a normal distribution is given by 5

$$P(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right).$$

Solve the equation

$$P(x) = \frac{1}{2}\ P\ (0)$$

for x to an accuracy of 10^{-5}. Interpret your answer.

3. Attempt any ***two*** of the following :

(a) Write an algorithm to integrate a function tabulated at n points using Simpson's rule. 2.5

(b) Derive two point Gauss quadrature formula. 2.5

(c) Write an algorithm to find the volume of a unit sphere using Monte Carlo method. 2.5

4. Attempt any ***two*** of the following :

(a) The radial neutron flux in a spherical reactor of radius R is given by 5

$n(x) = A\,x \sin(\pi x)$

Where $x = r/R$. Calculate the total flux in units of A by evaluating numerically the integral

$$N = \int_0^1 n(x)\,dx$$

using Simpson rule with 10 points.

(b) Integrate $f(x) = 2x^3 - 3x^2 + 4x - 5$ from -2 to 4 using two point Gauss quadrature formula. 5

(c) Integrate the standard Gaussian function 5

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

from $x = -1$ to 1 using the formula

$$\int_{-1}^{1} f(x)\,dx = R_1 f(x_1) + R_2 f(x_2) + \ldots + R_6 f(x_6)$$

Where $R_1 = 0.17132, R_2 = 0.36076, R_3 = 0.46791, R_4 = R_3, R_5 = R_2, R_6 = R_1, x_1 = -0.93247, x_2 = -0.66121, x_3 = -0.23862, x_4 = -x_3, x_5 = -x_2, x_6 = -x_1$.

5. Attempt any ***two*** of the following :

(a) Show that the Gauss-Seidel method gives a converging solution to the simultaneous equations. 2.5

$$a_{11}x_1 + a_{12}x_2 = a_{13}$$

$$a_{21}x_1 + a_{22}x_2 = a_{23}$$

if the following condition is satisfied :

$$\left|\frac{a_{12}a_{21}}{a_{11}a_{22}}\right| < 1.$$

(b) Describe how to solve the initial value problem 2.5

$$\frac{dy}{dx} = f(x, y); \qquad y(x_0) = y_0$$

for $y(x)$ on the interval $a \le x \le b$ using Euler method.

(c) Write an algorithm to solve a differential equation using Runge-Kutta method of order 4. 2.5

6. Attempt any ***two*** of the following :

(a) Solve the modified radioactivity equation

$$\frac{dN}{dt} = -\lambda N + r$$ 5

using Euler method with a step size of 1 over the interval $t = 0$ to $t = 10$ for $\lambda = 0.1$ and $r = 10$. The initial condition is N (0) = 1000. Compare your results with the analytical solution

$$N(t) = N(0)e^{-\lambda t} + \frac{r}{\lambda}(1 - e^{-\lambda t})$$

(b) Solve the initial value problem

$$\frac{dy}{dx} = -xy, \; y(0) = \frac{1}{\sqrt{2\pi}}, \; 0 \le x \le 1$$ 5

Using Runge-Kutta method of order 4 with a step size of 0.25, compare your results with the analytical solution

$$y(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

(c) Solve the following equations by Gauss-Seidel procedure to an accuracy of 10^{-3} : 5

$$9x_1 + 2x_2 + 4x_3 = 20$$
$$x_1 + 10x_2 + 4x_3 = 6$$
$$2x_1 - 4x_2 + 10x_3 = -15$$

7. Attempt any ***two*** of the following :

(a) A radioactive sample has 'N' nuclei at time $t = 0$. The probability of decay is 'P'. Write an algorithm which calculates the number of nuclei left in the sample as a function of time. 5

(b) 'N' neutrons incident normally from a reactor onto a rectangular slab which is infinite in x-direction and of thickness 't' along y-direction. The neutrons travel a distance of 1 unit and then are scattered repeatitively in random directions. A neutron gets absorbed in the slab after suffering 'm' scatterings. Write an algorithm to calculate (i) how many neutrons re-enter into the reactor, (ii) how many neutrons get absorbed in the slab, and (iii) how many of them passes through the slab.

(c) Explain how to solve the boundary value problem 5

$$y'' + p(x)y' + q(x)y = r(x),$$

$$a \le x \le b, y(a) = \alpha, y(b) = \beta,$$

using the finite difference method.

8 Attempt any *two* of the following :

(a) Write an algorithm to calculate the average distance 'd' travelled by a thermal neutron diffusing in a tank of heavy water by assuming that the neutron is performing a 3 dimensional random walk with a step size of 'h' of 'N' steps. 7.5

(b) Explain how to solve the boundary value problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y) \qquad 7.5$$

by finite difference method over the rectangular region

$$a \le x \le b, c \le y \le d$$

where, on the boundary of the region, we have

$$u(x, y) = g(x, y).$$

(c) The following boundary value problem 7.5

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,$$

$$0 \le x \le 1, 0 \le y \le 1,$$

$$u(x, 0) = 0, u(x, 1) = x,$$

$$u(0, y) = 0, u(1, y) = y.$$

is to be solved by finite difference method by dividing the x and y intervals into 5 equal parts. Find the linear simultaneous equations resulting from the finite difference method.