This question paper contains 3 printed pages.

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Your Roll No. ....

M. Tech. / II Sem.

A

NUCLEAR SCIENCE AND TECHNOLOGY

Paper NST 607— Accelerator Physics and Technology

Time : 3 hours

Maximum Marks : 70

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. 1 is compulsory and carries 16 marks. Attempt any ***three*** *from the remaining four questions, each one of which carries 18 marks.*

1. Attempt all parts.

(a) Consider two adjacent cells, each of length d, of an accelerating structure which operates in the π mode with electric field in the first cell given by $E_1 = E_0\sin(\omega t)$ and phase velocity v_{ph} (=ω/k) equals the speed of light c. Consider a particle with charge q entering in cell 1 at time t=0. Find the net energy gain by a particle in two cells if (i) particle's speed = c/5 and (ii) particle's speed = 2c/5. Symbols have their usual meanings. **[4]**

(b) Consider a lattice with unit FODO cell {QD/2 O QF O QD/2}. Calculate 2x2 transfer matrix for this FODO cell assuming equal strengths of focusing and defocusing quadrupoles and thin lens approximation. **[4]**

(c) Starting from the conservation of energy in a constant-Impedance RF cavity under steady-state condition, obtain expressions for the total accelerating voltage and filling time in terms of shunt impedance, quality factor, attenuation factor, input power, and length of the cavity (assuming negligible beam loading). **[4]**

(d) Consider a cyclotron operating with a magnetic field of 1.5 T and radius 0.5 m. Find the frequency of the alternating source which is needed to accelerate protons and the maximum energy (in eV) gained by these protons (assuming non relativistic limit). Take mass of the proton = 1.7×10^{-27} kg and proton charge = 1.6×10^{-19} C. **[4]**

2. (a) (i) Electrons are typically injected into an accelerating waveguide with relatively low velocities ($\beta_{el} < 1$). What would be minimum electric field amplitude of the RF wave E_Z^{min}(assuming sinusoidal variation) within the waveguide to capture a low energy electron? Take phase velocity of the RF wave v_{ph} to be equal to the speed of light, i.e. v_{ph}= c. **[7]**

(ii) An electron enters into a waveguide with initial kinetic energy of 30 keV. What would be the value of E_Z^{min} (in eV) if RF wave frequency of 2856 MHz is used? Take $v_{ph}= c$, electron mass =0.5MeV/c^2, electron charge =1 6 x 10^{-19}coulombs & c=3x10^8 ms^{-1}. [3]

(b) An electron is captured by a uniform waveguide and it starts moving with an almost uniform velocity v_{el}. Show that the necessary condition for electron acceleration in the waveguide is $v_{el} = v_{ph}$. Discuss briefly if a uniform wave-guide is suitable for electron acceleration or not. [4]

(c) Consider a cylindrical cavity with diameter 75 cm and length 50 cm. What would be resonant frequencies of the fundamental accelerating mode (TM_{010}) and the next highest mode (TM_{011}) if phase velocity v_{ph} = c (i.e. speed of light)? What kinetic energy of the protons would be required to ensure a transit time factor of 41% for TM_{010} mode of this cavity? First non zero root of the zeroth order Bessel function lies at 2.405 and c=3x10^8 ms^{-1} [4]

3. (a) Find an expression for the synchrotron oscillation frequency in phase-energy oscillations, assuming small amplitude case. Neglect the effect of the synchrotron radiation and take $\frac{\Delta\beta}{\beta} = \frac{1}{\gamma^2}\frac{\Delta p}{p}$ and $\frac{\Delta p}{p} = \frac{1}{\beta^2}\frac{\Delta E}{E}$, where symbols have their usual meanings. [6]

(b) A synchrotron accelerates protons to the kinetic energy of 10 GeV and has a magnetic field of 1.5 T.

(i) What are the values of the momentum (in GeV/c), total energy (in GeV) and the magnetic rigidity (in T-m)? Take mass of the proton = 0.94 GeV/c^2. [2]

(ii) What are the values of the bending radius and the total synchrotron radius if 2/3 of the circumference is filled with bending magnets? [1]

(iii) What would be revolution frequency of the proton? [2]

(iv) If voltage of the RF cavity seen by the proton is 7.8 keV and the synchronous phase is 135 degrees then what would be the peak RF voltage in the cavity? [1]

(v) Calculate phase slip factor if the momentum compaction factor is 0.25. [2]

(vi) If the harmonic number is 10, what is the value of the synchrotron oscillation frequency? [4]

4. (a) Starting from the Hill's equations in linear transverse beam dynamics, obtain transfer matrices (2x2) for the focusing and defocusing quadrupoles in the horizontal plane. [3]

(b) Starting from the trial solution of the Hill's equations (assuming quadrupoles only) and using Courant Snyder parameters $\alpha(s)$, $\beta(s)$, $\gamma(s)$, show that the phase space trajectory (x-x') of each particle of the beam satisfies the following relation:

$$\gamma(s)x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'^2(s) = \epsilon,$$

where symbols have their usual meanings. **[5]**

(c) Define r.m.s. beam emittance, beam matrix and beta matrix. Show that the evolution of beta matrix through an accelerating section is given by $B_1 = MB_0M^{-1}$, where M is the transfer matrix of the accelerating section and B_0 is the beta matrix at the starting point. **[4]**

(d) If Courant-Snyder parameters are given at the start and end of an accelerating section, find the transfer matrix M (2x2) in terms of these parameters and phase advance. What would be the transfer matrix assuming periodic conditions for Courant-Snyder parameters? How these parameters are related to the elements of transfer matrix? **[6]**

5. (a) (i) Starting from the relativistic form of the power radiated by an accelerated charge particle, deduce expressions for the radiated power and the energy loss per turn when charge particle has velocity perpendicular to acceleration. Find the ratio of the power radiated from an electron and that of a proton of the same energy. **[5]**

(ii) What is the power radiated from the bends by a 50 mA electron beam current stored in a circular accelerator, if the energy is 2 GeV and the bending radius is 50 m. [Hint: Power radiated by beam = No. of particles in a beam x Power radiated per particle.] Take $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m. **[4]**

(b) (i) Obtain an expression for the instantaneous luminosity at the interaction point for the electron-positron collision. Each beam consists of N_b equally spaced bunches circulating with a frequency f_{rev}. N_1 is the number of electrons in a bunch and N_2 is the number of positrons in a bunch. Assume same Gaussian distribution for both the beams. **[5]**

(ii) Calculate instantaneous luminosity (in $cm^{-2}s^{-1}$) at the interaction point for proton-proton collisions if beam sizes at the interaction point (IP) are $\sigma_X^* = 10\mu m$, $\sigma_Y^* = 20\mu m$, number of bunches per beam=3000, beam current = 0.5 A for each beam, and revolution frequency=10 kHz. **[4]**