This question paper contains 4 printed pages.

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Your Roll No. ....

M. Tech. / II Sem.

A

NUCLEAR SCIENCE AND TECHNOLOGY

Paper NST 609— Nuclear and Computational Science

Time : 3 hours | *Maximum Marks* : 70

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Question No. 1 is compulsory. Attempt any **four** from the rest.*
Symbols have their usual meanings.

1. Attempt any *seven*. **(7x2=14)**

(i) What happens in a while loop if the control condition is false initially? What is wrong with the following loop? **while (n<=100) sum += n*n;**

(ii) Write the output of the following program:

```
int main( )
{ long m, d, n=0;
 m= 52;
while (m>0)
      { d = m%10;
        m /= 10;
        n = 10*n + d;
      }
 cout<<"n = " <<n <<endl;}
```

(iii) What is the result of the following expression? Consider all the three scenarios; viz, x < y, x = y & x > y. **(x < y ? -1 : (x == y ? 0 : 1));**

(iv) What values will the elements of an array have when it is declared without its initializer? What happens if an array's initializer has more values than the size of the array?

(v) Explain that there cannot exist bound states in p-p & n-n system. (Assuming that a bound state in two-nucleon system can exist only in $\ell = 0$ & $s = 1$ state).

(vi) Predict the allowed values of J^π for the nuclei ${}^{18}_{9}F$ & ${}^{12}_{5}B$. (Hint: Nordheim's rule)

(vii) How can the neutron have a non-zero magnetic moment? How one can interpret the lack of exact equality of μ and $\mu_p + \mu_n$ in deuteron?

(viii) Write a single C++ statement using conditional expression operator to assign absolute value of **x** to a variable **absx**.

2. (a) The deuteron is a bound state of a proton and a neutron of total angular momentum J = 1. It is known to be principally an S (l = 0) state with a small admixture of a D (l = 2) state. Explain why a P & G state cannot contribute?

(b) Assuming square well potential show that deuteron cannot have an excited state.

Turn over

(c) The deuteron has magnetic moment ($\mu = 0.857\mu_N$) which is approximately the sum of proton and neutron magnetic moments ($\mu_p = 2.793\mu_N$, $\mu_n = -1.913\mu_N$), where μ_N is the nuclear magneton. From these facts what one can infer concerning the contribution of orbital & spin angular momentum of the neutron and proton in the deuteron?

(d) Following program is the series expansion of the **sqrt(1+x)** for $|x| < 1$. The program contains both the syntax and logical errors. First column corresponds to the line number. Write down the line number of the line that contains error, the original and the corrected version of the line.

$$\sqrt{1+x} = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{(1-2n)n!^2 4^n} x^n \text{ for } |x| < 1$$

```
1    #include<iostream>
2    #include<math.h>
3    #define acc=1.0e-6
4    float sersum(float x);
5    {
6    float t,s,r;
7    int i;
8    t = 1.0;
9    s = 1.0;
10        i = 1;
11        do
12        {
13            r = (-2*i)*(2*i-1)*(3-2*i) / ( pow(i,2)*(1-2*i)*4 );
14            t *= r;
15            s += t;
16            i += 1;
17        } while(fabs(t/s) < acc);
18        return(s);
19   }
20    main()
21   {
22        float x;
23        cout << "x\t" <<"SerSum" <<endl;
23        for (x=0; x < 1.0; x+=0.1)
24        {
25            cout<<x <<"\t" << sersum(x) <<endl;
26        }
27   }
```

(2,6,2,4)

3. (a) Show that the total scattering cross-section for the low energy n-p scattering is given by

$$\sigma_{sc} = \frac{4\pi \sin^2\delta_0}{k^2}$$

(b) Further, determine the phase-shift δ_0, & show that the total cross-section (σ_{sc}) can be written as

$$\sigma_{sc} \approx 4\pi(\frac{1}{\gamma} + r_0)^2,$$

where $\gamma = \sqrt{\frac{MW}{\hbar^2}}$, M being the average mass of the n and p, W being the binding energy of the bound n-p system and r_0 is the range of the nuclear force.

(c) What was the discrepancy in the observed vs. theoretical σ_{sc} in the n-p scattering? How was this discrepancy resolved?

(d) Assume a deuteron like hypothetical bound n-p system with binding energy of the triplet state (W_t) is 5 MeV and range of the nuclear force (r_0) is 1 F. If σ_{sc} is 40 barn then calculate σ_t & σ_s (in barn). (Take mass of neutron = mass of proton = 940 MeV/c^2, and ħc = 197 MeV-F).

(5,3,3,3)

4. (a) In the harmonic oscillator Shell Model, the energy levels are given by

$$E_{nl} = \hbar\omega(2n + l - 1/2) = \hbar\omega(\Lambda + 3/2),$$

where $\Lambda = 2n + l - 2$. Find the number of nucleons(N_Λ) in each shell & the total number of particles (accumulating) for all levels up to Λ_m, permitted by Pauli's exclusion principle.

(b) The magnetic moment μ_z of a nucleus is given by $\mu_z = (g_l l_z + g_s S_z)\mu_N$. Show that the magnetic moment of the odd-even nucleus with odd number of protons is given by

$$\langle\mu_z\rangle = \mu_N \left\{j - \frac{2.3\, j}{j+1}\right\}; \; with\, j = l - 1/2$$

(Given: $g_l(p) = 1$, $g_s(p) = 5.6$)

(c) Estimate the value of magnetic moment for $^{15}_{7}N$ & $^{39}_{19}K$ in terms of μ_N.

(d) Using the extreme particle shell model, predict the ground state spin & parity of the nuclei: $^{21}_{20}Ca, ^{23}_{11}Na\ ^{17}_{8}O$.

(4,4,3,3)

5. (a) How the observed continuous energy spectrum of electron in the beta-decay was explained?

(b) Using the Fermi theory, derive an expression for the number of electrons emitted during beta-decay with momentum between p & p + dp. Also plot the expected momentum distribution.

(c) What was the need to incorporate Fermi function in the Fermi theory of beta decay?

(d) What is meant by "forbidden transitions" in the Fermi theory? Which of the following is Fermi, Gamow-Teller or mixed transitions (in case of forbidden, mention the degree of forbidden-ness)?

$$(i)\quad ^{14}_{8}O(0^+) \rightarrow {}^{14}_{7}N^*(0^+)$$
$$(ii)\quad ^{40}_{19}K(4^-) \rightarrow {}^{40}_{20}Ca(0^+)$$

(iii) $^3_1H\left(\frac{1}{2}^+\right) \rightarrow {}^3_2He\left(\frac{1}{2}^+\right)$

(2,6,2,4)

6. (a) The classical radius of the electron in CGS units is defined as $e^2/(m_ec^2)$, has the value of 2.8×10^{-13} cm. where m_e is the electron rest mass,. Use this fact to calculate the Coulomb repulsion energy (in MeV) between two protons at a distance of 1 F. Given rest mass of electron is 0.51MeV/ c^2. On the basis of this and assuming binding energy (B.E.) can be due to Coulomb repulsion energy only & that nucleons are uniformly distributed over the volume of the nucleus (sphere of radius R), estimate the B.E. for oxygen nuclei (Z=8, A=15).

(b) It is found that for the normalized charge distribution, given by

$$\rho(r) = \left(\frac{a^3}{8\pi}\right) \exp\,(-ar)$$

the form factor, F(q), is given by

$$(1 + \frac{q^2}{a^2\hbar^2})^{-2}$$

where "a" is some constant and rest of the symbols have their usual meanings. Consider electrons with momentum 500MeV/c scattered elastically through an angle of 20^0 by a lead nucleus $^{206}_{83}Pb$. What would be the reduction in Mott cross-section if we assume above charge Distribution. Use $a=1.5F^{-1}$ & hc = 197 MeV-F.

(c) What is the significance of scattering length? How total scattering cross-section can be written in terms of the scattering length?

(d) Explain the difference between the following two declarations.

```
int n1 = n;
int& n2 = n;
```

(6,4,3,1)