This question paper contains 4 printed pages.]

3406 *Your Roll No. ....*

M.Tech. / II Sem. **A**

NANO SCIENCE AND NANO TECHNOLOGY

Paper : NSNT-201 : Quantum Mechanics

Time : 3 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer any five from the 1st 6 questions and the 7th question is ***compulsory****.*

1. Consider a two level system where the kets $|\psi_1>$ and $|\psi_2>$ form an orthonormal basis . Now define a new basis - $|\phi_1>$ and $|\phi_2>$ by

$$|\phi_1>=\frac{1}{\sqrt{2}}(|\psi_1>+|\psi_2>)$$ and

$$|\phi_2>=\frac{1}{\sqrt{2}}(|\psi_1>-|\psi_2>)$$

An operator $\hat{O}$ represented in $|\psi_i>$ basis is given by

$$\hat{O}=\begin{bmatrix}1 & a\\ a & 1\end{bmatrix}$$

Find the representation of $\hat{O}$ in the new basis. Here 'a' is a constant. 6

[P.T.O.

2. Consider a one-dimensional system described by the Hamiltonian $H = \frac{P^2}{2m} + V(x)$

(a) Show that $[H, x] = -i\hbar\, P/m$. 2

(b) Find the expectation value of $\hat{P}$ in an eigen state of the given $\hat{H}$. 4

Hint : Use the result obtained in (a) to evaluate (b).

3. A particle of mass m is confined within an infinite 1-D well, between $x = 0$ and $x = L$. The eigen states of the Hamiltonian are given by

$$\langle x|\phi_n\rangle = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi n x}{L}\right)$$

Where $E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$ $n = 1, 2, ...$

Consider a situation where at time $t = 0$ the particle is in state $|\psi(t=0)\rangle = \frac{|\phi_1\rangle + |\phi_2\rangle}{\sqrt{2}}$

(a) Find the time-dependent $|\psi(t)\rangle$ 1

(b) Calculate the coordinate representation of $|\psi(t)\rangle$, i.e. find $\langle x|\psi(t)\rangle$. 1

(c) Check if $|\psi(t)\rangle$ comes back to $|\psi(t=0)\rangle$ after a certain time t_0. If so determine t_0. 4

4. Using the uncertainty relation $\Delta x \Delta p \geq \hbar/2$, estimate the ground state energy of 1-D harmonic oscillator.

$$\hat{H} = \frac{\hat{P}^2}{2m} + \frac{mw^2}{2}\hat{x}^2$$

Given $<x> = <P> = O$ for all eigen state of $\hat{H}$ and $\Delta x = \sqrt{<x^2> - <x>^2}$ and $\Delta P = \sqrt{<P^2> - <\hat{P}>^2}$.

Hint: Express expectation value of $\hat{H}$ in terms of Δx and ΔP and then use uncertainty relation and minimization condition. 6

5. In the simultaneous eigen basis of $\hat{L}^2$ and $\hat{L}_z$ which is represented as $|l, m>$, obtain the matrix representation of L_x and L_y operator for $l = 1$ case.

Given, $L_+ = L_x + i L_y$

$L_- = L_x - i L_y$

$L_\pm |l, m> = \hbar \sqrt{l(l+1) - m(m+1)}\ |1, m \pm 1>$

$L_z |l, m> = m\hbar\ |m>$

$L^2 |l, m> = l(l+1)\hbar^2 |m>$

And the basis is given by {I L, $m = 0, \pm 1$>} 6

6. (a) Show that, in 1st order time independent perturbation theory, the 1st order correction to the n^{th} eigen state is orthogonal to the eigen state itself. 4

(b) Consider a particle of charge 'e' and mass 'm' in a harmonic oscillator potential $V = \frac{1}{2} m w^2 x^2$. Now apply a small electric field of magnitude 'f' along the $+ve$ x-axis.

So. $H = H_0 + H'$

$$= \frac{\hat{P}^2}{2m} + \frac{1}{2} m w^2 \hat{x}^2 + ef\hat{x}$$

Treating the electric field perturbatively, calculate the energy correction to n^{th} eigen state in 1^{st} order perturbation theory.

Given : $$a = \sqrt{\frac{mw}{2\hbar}} \left(\hat{x} + \frac{i\hat{P}}{mw} \right)$$

$$a^{\dagger} = \sqrt{\frac{mw}{2\hbar}} \left(\hat{x} - \frac{i\hat{P}}{mw} \right)$$ 2

7. (a) Consider 1-D harmonic oscillator problem and apply variational method to estimate

The ground state energy.

Trial wave function $\psi_0(x, \alpha) = Ae^{-\alpha x^2}$

Given $\int_{-\infty}^{\infty} x^{2n} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}\ 1.3.5....(2n-1)/(2a)^n$. 5

(b) Given the information of ground state how do we obtain an upper bound to the 1^{st} excited state using variational method. 3