This question paper contains 3 printed pages.

2229

Your Roll No. .....

M.A. / Winter Semester

A

ECONOMICS

Course 003— Introductory Econometrics

(Admissions of 1999 and onwards)

Time : 2 hours

Maximum Marks : 50

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do all questions, clearly showing your working. Start each answer on a fresh page Part A and Part B must be answered on separate booklets.

Part A

Some information that may or may not be useful to you for this part:

- For r≠1 the sum of the first *n* terms of a geometric series is:
  $a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} = \sum_{k=0}^{n-1} ar^k = a\frac{1-r^n}{1-r}$,
  where *a* is the first term of the series, and *r* is the common ratio.
- The sum of the first *n* natural numbers is: $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$

(1) A college team plays 10 football games during a season. In how many ways can it end the season with five wins, four losses, and one tie? (3)

(2) If the joint probability density of X and Y is given by
$f(x,y) = 2 \; for \; x > 0, y > 0, x + y < 1$
$f(x,y) = 0 \; otherwise$
find, Pr(X > 2Y). (5)

(3) Consider a random sample $(X_1, X_2, ..., X_n)$ where X_i follows the Bernoulli distribution with probability of success, $p = 0.5$. We would like the sample mean to lie within 0.1 of the population mean with probability 0.8. Without using any tables, find the minimum sample size that would achieve this objective. (5)

Turn over

(4) (a) The m.g.f. for a chi-squared random variable Y with n degrees of freedom is given by
$E(e^{tY}) = (\frac{1}{1-2t})^{n/2}, \ t < 1/2$
Using the m.g.f, calculate E(Y) and Var(Y). (3)

(b) Also, using the m.g.f, show that if $X_1, X_2, \ldots, X_k$ are independent random variables, and if X_i follows chi-squared with n_i degrees of freedom, then $X = X_1 + X_2 + \cdots + X_k$ follows chi-squared with $n_1 + n_2 + \cdots + n_k$ degrees of freedom. (3)

(5) Suppose that four letters arrive in your morning mail, but unfortunately one of them is misplaced before you have a chance to open it. If, among the remaining three letters, two contain credit-card billings and the other one does not, what would be the maximum likelihood estimate of k, the total number of credit card billings among the four letters received? (6)

Part B

(6) Show that the limits of R^2 may not be 0 and 1 in a regression model without an intercept term. (3)

(7) In the simple linear regression $Y_i = \beta_1 + \beta_2 X_i + u_i$, show that the coefficient of determination is equal to the square of the correlation between the actual and estimated Y–values. (3)

(8) Explain how you would test the assertion that the OLS estimator of the error variance is unbiased in a simple regression model, using a Monte Carlo experiment. (3)

(9) In the simple linear regression $Y_i = \beta_1 + \beta_2 X_i + u_i$, where the errors are heteroscedastic, explain why the OLS estimator of the slope coefficient is not likely to have minimum variance. (3)

(10) In the simple linear regression $Y_i = \beta_1 + \beta_2 X_i + u_i$, where the errors follow an autoregressive scheme of order 1, explain how you would use the iterative Prais-Winstem method to estimate the parameters. (3)

(11) In a simple regression of Y on X where both variables are subject to measurement errors, show that the slope coefficient can only be identified upto a range. (10)