

M.A. / Winter Semester

A

## ECONOMICS

Course 101— Mathematical Economics

(Admissions of 1999 and onwards)

Time : 2 1/2 hours

Maximum Marks : 70

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer any three of the following questions.

Each question is worth 23 1/3 marks; marks for each part of a question are indicated at the end of the question.

- Q.1 Consider a household with two members (1 and 2), each of whom gets utility from consuming a good  $c$  and leisure  $l$ . The household's problem then is to maximize its utility over its budget constraint, as below:

$$\begin{array}{l} \text{Max. } U(c_1, c_2, l_1, l_2) \\ \text{Subject to } p(c_1, c_2) + w_1 l_1 + w_2 l_2 \leq Y + T(w_1 + w_2) \end{array}$$

where  $p$  is the price of  $c$ ,  $w$  is the wage rate,  $Y$  is the household's non-labor income and  $T$  is the individual time endowment.

- a. Write down the test you would conduct for the validity of this model in terms of household resource allocation decisions. Point out two confounding factors in empirically testing the model's validity. [10 points]

Suppose the utility function of the household is now replaced by:

$$\sum_i \lambda_i U_i(c_i, l_i)$$

where  $i=1, 2$  and  $\sum \lambda_i = 1$

- b. Are the implications of this model for the household's resource allocation process different from part a? Explain your answer. [5 points]

- c. Compare the implications for Pareto efficiency of resource allocation within households in the two models. [5 points]

Q.2. Suppose households have the following utility function:

$$U = U(c, e_1, e_2, b_1, b_2)$$

or

$$= V[c, W(e_1, e_2, b_1, b_2)]$$

$c$  is parental consumption,  $e_i$  is life time earnings and  $b_i$  is the parental bequests to child  $i$ .  $W$  is the sub-utility function over  $e$  and  $b$ . Assume parents exhibit "equal concern."

Now, suppose parental preferences over  $e$  and  $b$  for two households (I and II) can be expressed, respectively, as:

$$\text{I. } W^*[W^E(e_1, e_2), W^B(b_1, b_2)]$$

$$\text{II. } W[(e_1 + rb_1), (e_2 + rb_2)]$$

where  $r$  is the rate of interest.

Parents maximize  $W(\cdot)$  s.t.:

$$\sum_{i=1}^2 b_i = I^b$$

$$P_s \sum_{i=1}^2 S_i + \sum_{t=1}^T P_t \sum_{i=1}^2 X_{it} = I^e$$

$P$  is price and  $I$  is household income allocated to bequests ( $b$ ) and earnings potential ( $e$ ).

- a. Derive the implications of the model for distribution of bequests between child 1 and 2 for household I from the sub-utility maximization process. [5 points]
- b. In household I, what does the maximization of sub-utility function,  $W(\cdot)$  imply for allocation of household resources to schooling ( $S$ ) between child 1 and 2, if:  
 $e_i = e(G_i, S_i, X_i)$  (in both households;  $G$ =genetic endowments,  $S$ = schooling,  $X$ =other goods) and  $G_1 > G_2$ ? [10 points]
- c. Compare your results in part b to the implications of preferences of household II for allocation of household resources to schooling between child 1 and 2 if  $G_1 > G_2$ . [5 points]

Q.3. Refer to the attached Figures (5 and 6) from the assigned course paper:

Qian, Nancy. 2008. "Missing Women and the Price of Tea in China: The Effect of Sex Specific Income on Sex Imbalance." *Quarterly Journal of Economics* 123(3).

- a. Briefly, describe the natural experiment the author utilizes to identify the impact of sex specific income on human capital outcomes, by gender. [7 points]
- b. Does the empirical evidence shown in this paper support rejection of the unitary framework of the household? Explain with specific reference to the results for sex-ratios and education. [8 points]

Q.4. Propose a research agenda on any of the topics covered in the course syllabus.

- a. State and motivate your research question (100 words max.) [6 points]
- b. Elaborate on the research innovation in you project in the context of the literature. (100 words max.) [9 points]