

2235

A

M.A. Winter Semester

ECONOMICS

Course 104 – Game Theory - II

(Admissions of 1999 & onwards)

Time : 2-1/2 hours

Maximum Marks : 70

(Write your Roll No. on the top of immediately on receipt of this question paper).

Attempt as many as you want.

1.
 - a) Define a TU game and an allocation of a TU game. [3]
 - b) Define Symmetry, Dummy and Additivity in this context. [3]
 - c) Define the Shapley value. [3]
 - d) Show that the Shapley value satisfies the above properties. [5]
 - e) Propose an allocation which satisfies Symmetry and Additivity but violates Dummy. [3]
 - f) Define a convex game. [3]
 - g) A game (N, v) is called superadditive if for all S, T such that $S \cap T = \emptyset$, $v(S \cup T) \geq v(S) + v(T)$. Show that convex games are also superadditive. [5]
 - h) Give an example of a superadditive game which is not convex. [5]
2. Consider an auction setting. Suppose there are just two bidders and their valuations are independently drawn from a uniform distribution on $[0, 1]$. Consider a second price auction.
 - a) Show that truth telling is a dominant strategy? [2]
 - b) Show that truth telling is a Bayes-Nash equilibrium? [4]
 - c) Find the seller's expected revenue. [4]
 - d) Define a second price auction with a reserve price. [2]
 - e) Show that truth telling is still a Bayes-Nash equilibrium? [2]
 - f) Find the seller's expected revenue. [4]
 - g) Can you use the 'revenue equivalence principle' to compare (c) and (f)? Why or why not? [2]
3. Consider a bilateral trade setting in which buyer's and seller's valuations (θ_B and θ_S respectively) of an indivisible object are drawn independently from the uniform distribution on $[0, 1]$. Utility of buyer and seller are as follows, $u_i = \theta_i \cdot 1_i + \text{transfer to } i$, where $1_i = 1$ if i gets the object and 0 otherwise. Suppose that θ_i is private information of agent i .
 - a) Model this situation as a mechanism design problem (specify the Bayesian game) [3]
 - b) Define efficiency, individual rationality and budget balancedness in this context? [3]
 - c) Find the 'pivotal' mechanism. [3]
 - d) Is 'pivotal' mechanism individually rational? [3]
 - e) Find a mechanism which is Bayesian incentive compatible (where truth telling is Bayesian-Nash equilibrium), efficient and individually rational. [5]
 - f) Show that if a mechanism is Bayesian incentive compatible, efficient and individually rational then the sum of the buyer's and seller's expected utilities can not be less than $\frac{5}{6}$. [5]
 - g) Show that if a mechanism is budget balanced then the sum of the buyer's and seller's expected utilities can not exceed $\frac{5}{6}$. [5]
 - h) Show that there does not exist any Bayesian incentive compatible, efficient, individually rational and budget balanced mechanism in bilateral trading. [3]