This question paper contains 4 printed pages.

3097 *Your Roll No.* ....

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Paper— EE.502

COMPUTATIONAL MATHEMATICS

Time : 3 hours *Maximum Marks* : 100

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all the questions by selecting any two parts from each. Each question carries equal marks.

1. (*a*) Show that:

(*i*) $\Delta(e^{2x} \log 3x) =$

$$e^{2x}\left[e^2 \log\left(1+\frac{1}{x}\right)+(e^2-1)\log 3x\right].$$

(*ii*) $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1+\frac{1}{4}\delta^2}$.

(*b*) Prove that:

(*i*) $$xy_1 + x^2y_2 + x^3y_3 + \ldots = \left(\frac{x}{1-x}\right)y_1 + \left(\frac{x}{1-x}\right)^2 \Delta y_1 + \left(\frac{x}{1-x}\right)^3 \Delta^2 y_1 + \ldots$$

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(*ii*) Express the function $f(x)=2x^3+3x^2-5x+4$ and its successive differences in factorial notation. Also obtain a function whose first difference is $f(x)$.

(*c*) Define interpolation. Derive Bessel's central difference interpolation formula from Newton's forward interpolation formula and hence find a polynomial of degree three or less which takes the following values of the function $f(x)$:

x :	4	6	8	10
$f(x)$:	1	3	8	20

2. (*a*) Use Everett's formula to obtain $f(1\cdot 15)$ given that $f(1)=1\cdot 000$, $f(1\cdot 10)=1\cdot 049$, $f(1\cdot 20)=1\cdot 096$, $f(1\cdot 30)=1\cdot 140$.

(*b*) The population of a certain town is shown in the following table:

Year :	1921	1931	1941	1951	1961
Population (in thousands) :	19·96	38·65	58·81	77·21	94·61

Estimate the population in the year 1936. Also find the rate of growth of population in 1951.

(*c*) From the following table, find x, correct upto two decimal places, for which y is maximum and find this value of y :

x :	1·2	1·3	1·4	1·5	1·6
y :	0·9320	0·9636	0·9855	0·9975	0·9996

3. (*a*) Find the condition for convergence of Newton-Raphson method and show that this method has a quadratic convergence.

   (*b*) Solve the following system by Crout's method:

$$2x-3y+10z=3, \qquad -x+4y+2z=20,$$
$$5x+2y+z=-12.$$

   (*c*) Discuss Power method for finding the largest eigenvalue of a matrix and hence determine the largest eigenvalue and the corresponding eigen-vector of the matrix

$$A=\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

4. (*a*) A solid of revolution is formed by rotating about the x-axis the area between the x-axis, the lines $x=0$ and $x=1$ and a curve through the points with the following coordinates:

x :	0·00	0·25	0·50	0·75	1·00
y :	1·000	0·9896	0·9589	0·9089	0·8415

   Estimate the volume of the solid formed, giving the answer to three decimal places.

(*b*) Use Picard's method to approximate y when $x=0\cdot1$, given that

$$\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+y=0 \quad \text{and} \quad y=0\cdot5,$$

$$\frac{dy}{dx}=0\cdot1 \quad \text{when } x=0.$$

(*c*) Find the error in Simpson's one third rule and Weddle's rule.

5. (*a*) By Runge-Kutta method of fourth order, solve the simultaneous first order differential equation

$$\frac{dy}{dx}=yz+x, \quad \frac{dz}{dx}=xz+y$$

given that $y(0)=1$, $z(0)=1$ for $y(0\cdot1)$ and $z(0\cdot1)$.

(*b*) Solve $u_{xx}+u_{yy}=0$ in $0\le x\le 4$, $0\le y\le 4$ given that $u(0, y)=0$, $u(4, y)=8+2y$, $u(x, 0)=x^2/2$ and $u(x, 4)=x^2$. Take $h=k=1$ and obtain the result correct to one decimal.

(*c*) Solve $u_{xx}+u_{yy}=8x^2y^2$ for a square grid with $h=1$ and $u(x, 0)=0$ on the boundary.

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