[This question paper contains 5 printed pages.]

Your Roll No. ....

3213 **J**

MEE

Paper – EE.558

DIGITAL SIGNAL PROCESSING

Time : 3 *Hours* *Maximum Marks* : 100

(*Write your Roll No. on the top immediately on receipt of this question paper.*)

Attempt any ***five*** *questions.*

All *questions carry equal marks.*

1. (a) Define the energy signal and power signal and find the energy and power of periodic signal. 3

   (b) Prove that the energy of a real valued energy signal is equal to the sum of the energies of its even and odd components. 2

   (c) A discrete time system can be : 6

   (i) Linear or non-linear

   (ii) Time-variant or time-invariant

   (iii) Stable or unstable

   with respect to above properties, examine the following systems :

   1. $y(n) = x(-n+1)$

   2. $y(n) = x(n) \sin \omega_0 n$

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(d) Derive an expression for the convolution of two discrete-time sequences. 3

(e) Determine the output of a linear time-invariant system with impulse response : 6

$$h(n)=\left\{\underset{\uparrow}{6},\ 5,\ 4,\ 3,\ 2,\ 1\right\}$$

when the input signal is :

$$x(n)=\left\{\underset{\uparrow}{1},\ 1,\ 1,\ 1,\ 1\right\}$$

2. (a) Determine the discrete-time Fourier transform of the following signals : 10

$$n\ x(n).$$

where

(i) $x(n)=\begin{cases}1-\frac{1}{2}n & |n|\leq 3\\ 0 & \text{otherwise}\end{cases}$

(ii) $x(n) = u(n) - u(n-8)$

(b) Determine the signal whose Fourier transform is given by : 8

(i) $X(\omega) = \cos^2 \omega$

(ii) $X(\omega) = \begin{cases}0 & 0\leq|\omega|\leq\omega_0\\ 2 & \omega_0\leq|\omega|\leq\pi\end{cases}$

(c) If $x(n) \xleftrightarrow{F} X(\omega)$ 2

then prove that :

$$e^{j\omega_0 k} x(n) \leftrightarrow X(\omega - \omega_0)$$

3. (a) Prove that circular convolution of two finite length sequences in time domain is multiplication of two sequences in Frequency domain. 6

(b) Show that : 4

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{n=0}^{N-1} |X(K)|^2$$

(c) Given a sequence : 5

$$x(n) = \left\{\underset{\uparrow}{1}, 2, 3, 4\right\}$$

Compute its DFT X(K).

(d) DFT of a sequence $x(n)$ is given by : 5

X(K) = {10, − 2 + 2j, − 2, − 2 − 2j}

Determine the sequence $x(n)$.

4. (a) Given a sequence : 5+5

$$x(n) = \left\{\underset{\uparrow}{1}, 1, -1, -1\right\}$$

Evaluate its DFT using :

(i) decimation-in-time algorithm

(ii) decimation-in-frequency algorithm.

(b) Explain decimation-in-time algorithm. 10

5. (a) Explain the IIR filter design by impulse invariance method and develop the transformation. 8

(b) Design a digital Lowpass Butterworth Filter using impulse invariance method to meet the following specifications : 12

Passband edge frequency = 1.25 kHz

Stopband edge frequency = 2.75 kHz

Passband ripple $\leq$ 0.5 dB

Stopband attenuation $\geq$ 15 dB

Sampling frequency = 10 kHz

Derive the formulas used in above design.

6. (a) Given a second-order transfer function : 14

$$H(z) = \frac{0.5(1 - z^{-2})}{1 + 1.3z^{-1} + 0.36z^{-2}}$$

Perform the filter realization using the following realization :

(i) direct form–I and II.

(ii) cascade form via first order section

(iii) parallel form via first order section.

(b) Obtain a cascade realization using minimum numbers of multiplier for the system : 4

$$H(z) = \left(1 + \frac{1}{2}z^{-1} + z^{-2}\right)\left(1 + \frac{1}{4}z^{-1} + z^{-2}\right)$$

(c) Compare the FIR and IIR filter. 2

7. (a) For a Band pass filter, desired response is given by : 15

$$H_d(\omega) = \begin{cases} e^{-j\omega n_0} & \omega_{c_1} \le |\omega| \le \omega_2 \\ 0 & \text{otherwise} \end{cases}$$

where $\omega_{c_1} = \frac{\pi}{4}$ and $\omega_{c_2} = \frac{\pi}{2}$

Design a band pass filter using :

(i) Rectangular window

(ii) Hann window

(iii) Hamming window

(b) Prove the circular time shift property of DFT. 5