This question paper contains 4 printed pages.

3096 *Your Roll No.* ....

MEE J

Paper— EE . 501

ADVANCED MATHEMATICS

Time : 3 hours ***Maximum Marks*** : 100

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any five questions.
All questions carry equal marks.

1. (*a*) In a test, an examinee either guesses, or copies or knows the answer to multiple choice questions with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copied it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question, given that he correctly answered.

   (*b*) Define probability density function of a continuous random variable. The diameter x of an electric cable is assumed to be a continuous variate with possible probability density function

$$f(x) = 6x(1-x), \quad 0 \le x \le 1.$$

Verify whether $f(x)$ is probability density function. Also find the mean and variance.

2. (*a*) Find the probability distribution of the number of green balls drawn when three balls are drawn one by one without replacement from a bag containing three green and five white balls.

   (*b*) Define stochastic processes. Explain its different classifications with examples.

3. (*a*) Show that R^+, the set of all positive real numbers, is a vector space over R under the following operations:

$$u + v = uv$$
$$\alpha u = u^\alpha, \qquad \forall \alpha \in R,\ u, v \in R^+.$$

   (*b*) Give an example to show that union of two subspaces of a vector space need not be a subspace. Also show that union of two subspaces of a vector space is a subspace iff one is subset of the other.

4. (*a*) Define linearly independent vectors. Verify whether the vectors (1, 1, 1), (1, 2, 3) and (0, 1, 2) are linearly independent.

(*b*) Define basis and dimension of a vector space. Find basis and dimension of a subspace of R^3 generated by

$$S=\{(1,-3,2),(5,0,4),(2,-6,4)\}.$$

5. (*a*) Define a linear transformation, its null space and range space. Also prove that null space and range space form vector spaces.

(*b*) If $T: U(F) \to V(F)$ is a linear transformation, where U(F) is a finite dimensional vector space, then show that:

$$\text{Rank}(T) + \text{Nullity}(T) = \text{dim. } U.$$

6. (*a*) Find the eigenvalues and eigenvectors of the transformation,

$$T(x_1, x_2)=(4x_1+2x_2, 3x_1-x_2).$$

(*b*) Write a short note on the existence and uniqueness theorems for solution of an initial value problem. Give specific examples where the conditions are satisfied / not satisfied.

7. (*a*) Prove that the necessary condition for

$$I=\int_{x_1}^{x_2} f(x, y, y')\,dx$$

to be extremum is:

$$\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0.$$

(*b*) Find the geodesics on a sphere of radius *k*.

8. (*a*) Find the plane curve of fixed perimeter and maximum area.

(*b*) Find the cxtremal of the functional

$$\int_0^{\pi/2}(y'^2-y^2+2xy)\,dy,$$

given that $y(0)=0$ and $y\left(\frac{\pi}{2}\right)=0$.

*