This question paper contains 4 printed pages.

3107 *Your Roll No.* .....

MEE **J**

Paper – EE.602

NON-LINEAR CONTROL THEORY

Time : 3 hours *Maximum Marks* : 100

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. 1 is compulsory.
Answer any four questions from remaining.

1. (*a*) Explain the following w.r.t. non-linear system:

(*i*) Chaos

(*ii*) Bifurcation

(*iii*) Limit cycle

(*iv*) Jump resonance. 10

(*b*) Show that following systems are non-linear:

(*i*) $Y=5X+8$

(*ii*) $\ddot{Y}+5Y=5X+6$. 4

(*c*) For given non-linear system

$$\dot{X}_1=X_2$$

$$\dot{X}_2=\left(0\cdot1-\frac{10}{3}X_2^2\right)X_2-X_1+X_1^3$$

determine the equilibrium points. 6

P. T. O.

2. (*a*) Describe in detail different types of singularities of non-linear system. 10

(*b*) Develop the describing function for the non-linearity shown in fig. 1 10

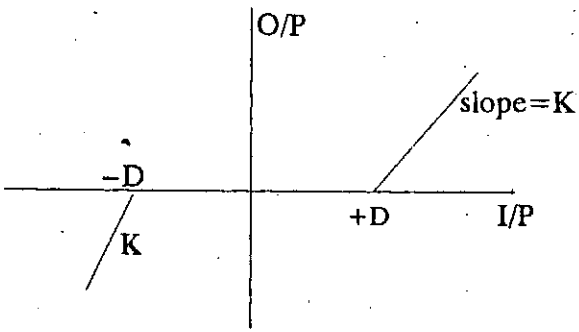
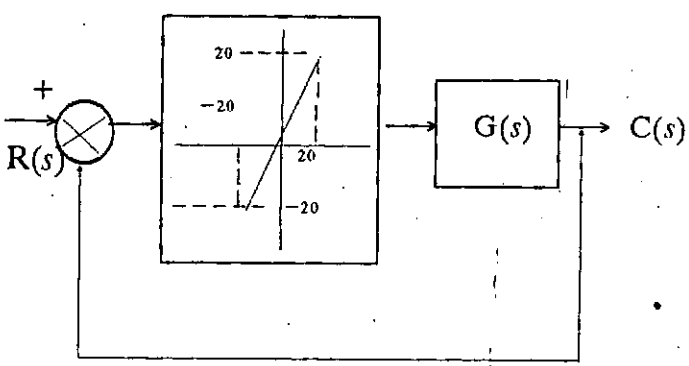


Fig. 1

3. Consider the non-linear system of fig. 2. Determine the largest K which preserves the stability of the system. If $K=2K_{max}$, find the amplitude and frequency of the self-sustained oscillations.



where $G(s)=\dfrac{K}{s(1+0\cdot 1s)(1+0\cdot 02s)}$ 20

4. (*a*) Verify that $V=X_1^4+X_1^2+2X_1X_2+2X_2^2$ is a suitable Liapunov function for the system described by:

$$\dot{X}_1=X_2$$

$$\dot{X}_2=-X_2-X_1^3.$$ 10

(*b*) Explain the concept of absolute stability in non-linear systems. State and explain Popov's criterion of stability. 10

5. (*a*) Use variable gradient method to find a Liapunov function for non-linear system:

$$\dot{X}_1=-X_1$$

$$\dot{X}_2=-X_2+X_1X_2^2.$$ 10

(*b*) Apply Krasovskii method to access the stability of the equilibrium point $X=0$ of the system given by

$$\dot{X}_1=-X_1$$

$$\dot{X}_2=X_1-X_2-X_2^3.$$ 10

6. (*a*) Draw the phase trajectory of the following system using isocline method:

$$\ddot{X}+0\cdot 5\dot{X}+2X+\dot{X}^2=0$$

with $X(0)=2, \quad \dot{X}(0)=0.$ 10

P. T. O.

(*b*) Investigate the stability of the system described by:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}. \qquad 10$$

7. (*a*) Construct phase plane trajectory using δ-method for the non-linear differential equation :

$$\ddot{X}+\dot{X}+X^2-1=0$$

with initial condition $X(0)=2$ and $\dot{X}(0)=0$. 10

(*b*) A linear autonomous system is described by $\dot{Y}=AX$

$$\text{where } A=\begin{bmatrix} -4K & 4K \\ 2K & -6K \end{bmatrix}$$

Find restriction on parameter K to guarantee stability of the system. Use Liapunov's direct method of stability. 10

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