[This question paper contains 5 printed pages.]

3100 *Your Roll No.* ....

MEE **J**

Paper – EE.551

SYSTEM THEORY

Time : 3 hours *Maximum Marks : 100*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer Q. No. **1** *which is compulsory and any other* **five** *questions.*

1. (a) Solve the difference equation :

$$X(K+2) + 3X(K+1) + 2\,X(K) = 0;$$

$$X(0) = 0,\ X(1) = 1.$$

(b) Obtain the Z-transform of $\frac{1}{S(S+1)}$.

(c) Consider the system

$$X(K+1) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} X(0),$$

Find the state transition matrix $\phi(K)$.

P.T.O.

(d) Find the diagonal system that is similar to the following system. Use diagonalization technique :

$$\dot{X}(t) = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t)$$

$$Y(t) = [2 \quad 3]\, X(t)$$

(e) A system is described by the following differential equation. Represent the system in phase variable form.

$$\frac{\partial^3 x}{\partial t^3} + 3\frac{\partial^2 x}{\partial t^2} + 4\frac{\partial x}{\partial t} + 4x = u_1(t) + 3u_2(t) + 4u_3(t)$$

and outputs are

$$Y_1 = 4\frac{\partial x}{\partial t} + 3u_1(t)$$

$$Y_2 = \frac{\partial^2 x}{\partial t^2} + 4u_2(t) + u_3(t)$$

(4+4+4+4+4=20)

2. Find the unit step response for the first-order temperature control system shown in Fig. 1.

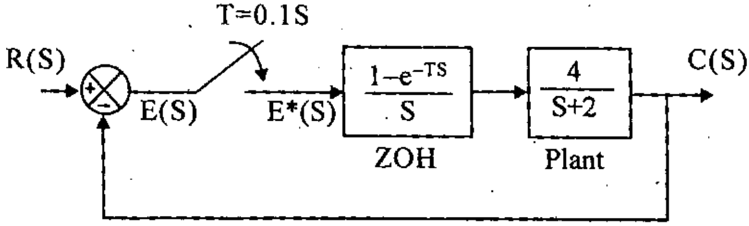

Fig. 1

Compare with the response of closed-loop continuous system with the same plant. (16)

3. Consider the system

$$\begin{bmatrix} x_1(K+1) \\ x_2(K+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1(K) \\ x_2(K) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (-1)^K$$

$x_1(0) = 1 = x_2(0)$

$y(K) = x_1(K)$

Find $Y(K)$ for $K \geq 1$. (16)

4. A discrete-time system has the transfer function

$$T(Z) = \frac{4Z^3 - 12Z^2 + 13Z - 7}{(Z-1)^2 (Z-2)}$$

Determine the state model of the system in

(i) Phase variable form

(ii) Jordan Canonical form (16)

5. Given $\dot{X}(t) = \begin{bmatrix} -7 & 1 \\ -12 & 0 \end{bmatrix} X(t) + \begin{bmatrix} 2 \\ -1 \end{bmatrix} u(t)$

and output $Y = [3 \quad -4]X(t) + [2]u(t)$

with the input $u(t) = 3e^{-t}$ and initial state vector

$$X(0) = \begin{bmatrix} -6 \\ 1 \end{bmatrix}.$$

Find system output Y(S) i.e. Laplace transform of Y(t). (16)

6. Consider the dynamics of a non-homogeneous system as

$$\dot{X}(t) = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

and output Y(t) [1 0] X(t), given the initial condition $X(0) = [1 \quad 0]^T$ where u(t) is a unit-step input.

Determine the output Y(t) at t = 1 sec. (16)

7. For a system represented by the state equation :

$$\dot{X}(t) = A\,X(t)$$

the response of

$$X(t) = \begin{bmatrix} e^{-2t} \\ -2e^{-2t} \end{bmatrix} \text{ when } X(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

and

$$X(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} \text{ when } X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Determine the system matrix A and the state transition matrix. (16)

8. Consider the system

$$\dot{X}(t) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u(t)$$

$$Y(t) = [1 \quad 0 \quad 2]\, X(t).$$

(a) Draw the block diagram. Determine controllability and Observability of each state variable.

(b) Find the eigenvalues of A and from these determine the stability of the system.

(c) Find the transfer function and from this determine the stability of the system. (6+5+5)

9. (a) Find the transfer function of armature controlled dc motor.

(b) Find the response X(K) of the following system :

$$X(K+2) - 3X(K+1) + 2X(K) = u(K)$$

where $X(K) = 0$ for $K \le 0$

$u(0) = 1$

$u(K) = 0$ for $K < 0, \quad K > 0$ (8+8)