This question paper contains **4+2** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2441**

Unique Paper Code : **2362801** **F-4**

Name of the Paper : **Optimization Technique**

Name of the Course : **B.Sc. (H) Statistics : Allied Course**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory.

Attempt *five* questions in all.

Use of non-scientific calculator is permitted

1. (*a*) Define a convex function. Test whether the following function is convex :

$$f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 - x_2x_3 - x_1 - 2x_3, (x_1, x_2, x_3) \in R^3.$$

(*b*) Use Dynamic Programming technique to find the minimum value of :

$$z = y_1^2 + y_2^2 + y_3^2$$

subject to :

$$y_1 \cdot y_2 \cdot y_3 = c \text{ and } y_j \geq 0, j = 1, 2, 3.$$

P.T.O.

(*c*) A manufacturer makes a product, of which the principal ingredient is a chemical X. At the moment, the manufacturer spends Rs. 1,000 per year on supply of X. but there is a possibility that the price may soon increase to four times its present figure because of a worldwide shortage of the chemical. There is another chemical Y, which the manufacturer could use in conjunction with a third chemical Z, in order to give the same effect as chemical X. Chemicals Y and Z would together cost the manufacturer Rs. 3.000 per year, but their prices are unlikely to rise. Using maximum and minimax criterion respectively, what decision the manufacturer should take ?

(*d*) Describe Goal Programming Problem and discuss its applications.

(*e*) Obtain Kuhn Tucker necessary conditions for the following NLPP :

$$\text{Max } Z = 8x_1 + 10x_2 - x_1^2 - x_2^2$$

subject to :

$$3x_1 + 2x_2 \leq 6 \text{ and } x_1, x_2 \geq 0.$$

5×5=25

2. A 4-ton vessel can be loaded with one or more of the three items. The following table gives the unit weight w_i, in tons and the unit revenue in thousands of dollars, r_i, for item i. How should the vessel be loaded to maximize the total return ? 12½

Item i	Weight w_i	Revenue r_i
1	1	30
2	2	60
3	3	80

3. Use Wolfe's method to solve the following quadratic programming problem (QPP) : 12½

$$\text{Min } Z = x_1^2 - x_1x_2 + 2x_2^2 - x_1 - x_2$$

Subject to :

$$2x_1 + x_2 \leq 1, \quad x_1, x_2 \geq 0.$$

4. (*a*) A manager has a choice between (*i*) A risky contract promising Rs. 7 lakhs with probability 0.6 and Rs. 4 lakhs with probability 0.4, and (*ii*) A diversified portfolio consisting of two portfolio consisting of two contracts with independent outcomes promising Rs. 3.5 lakhs with probability 0.6 and Rs. 2 lakhs with probability 0.4 respectively. Construct a deicsion tree and determine a decision using expected monetary value (EMV) criteria. 6½

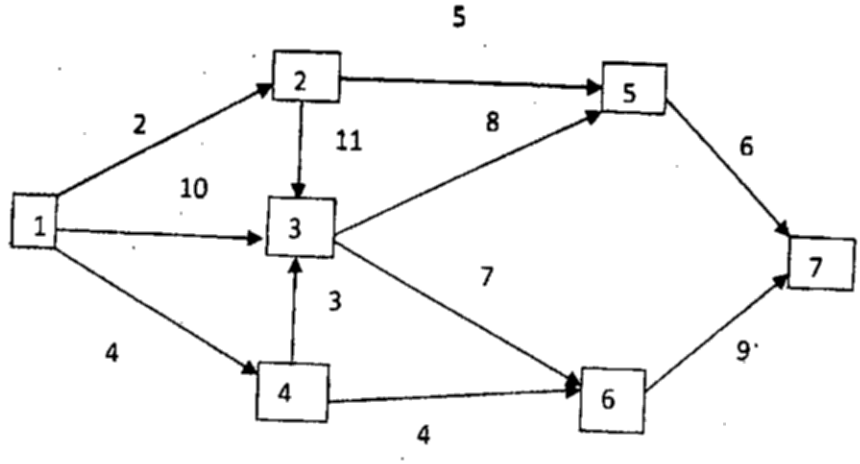
(*b*) The research department of ABC Company has recommended to launch a shampoo of three different types. The marketing manager has to decide one of the types of shampoo to be launched under the following estimated payoffs for various levels of sales :

Types of Shampoo	**Estimated Levels of Sale (Units)**		
	High	Medium	Low
Egg Shampoo	30	10	10
Clinic Shampoo	40	15	5
Deluxe Shampoo	55	20	3

What will be the marketing manager's decision if (*i*) Laplace, and (*ii*) Regret criteria are applied ? 6

5. (*a*) The following network provides the possible routes between the starting city at node1 and the destination city at node 7. The routes pass through intermediate cities designated

by nodes 2 to 6. Find the shortest route and the shortest distance from node 1 to node 7 using backward recursive equation. 6½



(*b*) A firm produces two products, say X and Y. Product X sells for a net profit of Rs. 80 per unit, while product Y sells for a net profit of Rs. 40 per unit. The goal of the firm is to earn Rs. 900 in the next week. Also, the management wants to achieve sales volume for the two products close to 17 and 15 respectively. Formulate this as a goal programming problem. 6

6. (*a*) Solve the following goal programming problem graphically :

Minimize $Z = \{d_1^+, d_2^-, d_3^-\}$ and satisfy the goals :

$$G1 : 20x_1 + 30x_2 + d_1 - d_1^+ = 80$$

$$G2 : 400x_1 + 400x_2 + d_2 - d_2^+ = 2000$$

$$G3 : 2x_2 + d_3 - d_3^+ = 14$$

$$x_j, d_i^-, d_i^+ \geq 0, \text{ for all } j = 1, 2, \text{ and } i = 1, 2, 3.$$

The goals have been listed in order of priority. 6½

P.T.O.

(*b*) Solve the following non-linear programming problem (NLPP) :

$$\text{Max } Z = 2x_1 - x_1^2 + x_2$$

Subject to

$$2x_1 + 3x_2 \leq 6$$

$$2x_1 + x_2 \leq 4$$

$$x_1, x_2 \geq 0.$$ 6

7. Write short notes on the following :

(*a*) Weights method and Preemptive method in goal programming.

(*b*) Convex and concave functions.

(*c*) The criteria of realism and regret in Decision Analysis. 4,4,4½