[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2436 **E** **Your Roll No................**

Unique Paper Code : 2352701

Name of the Course : **Allied Course – Physics**

Name of the Paper : Real Analysis

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** the questions are compulsory.
3. Attempt any **two** parts from each question.
4. Marks of each part are indicated.

1. (a) (i) For any pair of real numbers x and y, prove that

$$|x| - |y| \leq |x - y|. \quad (3)$$

(ii) Determine the set $A = \{ x : |x - 7| = |2x + 8| \}$. (4.5)

(b) State and prove Archimedean property of real numbers. (7.5)

(c) Find the least upper bound (l.u.b.) and greatest lower bound (g.l.b.) of each of the following sets :

(i) $A = \left\{ \frac{n}{n+1} : n \in N \right\}$.

(ii) $B = \left\{x^2 : x \in (-3,2)\right\}$.

(iii) $C = \left\{\frac{1}{n} : n \in N\right\}$. (7.5)

2. (a) Define a convergent sequence of real numbers and prove that every convergent sequence is bounded. Is the converse true ? Justify. (7.5)

(b) Find the limit of each of the following sequences $\{x_n\}$, where

(i) $x_n = \frac{(6-n)^2}{(1+6n)^2}$.

(ii) $x_n = \frac{\sqrt{2n^2+1}}{\sqrt{3n^2+1}}$.

(iii) $x_n = \frac{3n - 7\sqrt{n}}{n}$. (7.5)

(c) Define a Cauchy sequence. Prove that the sequence $\{x_n\}$, where

$$x_n = 1 + \frac{1}{3} + \frac{1}{5} + \ldots + \frac{1}{2n-1}$$

is not a Cauchy sequence. (7.5)

3. (a) Define continuity of a function at a point. Examine the continuity of the function

$$f(x) = \begin{cases} -x^2 & , \text{if } x \le 0 \\ 5x - 4 & , \text{if } 0 < x \le 1 \\ 4x^2 - 3x & , \text{if } 1 < x < 2 \end{cases}$$

at $x = 0, 1$. (7.5)

(b) State Lagrange's Mean Value theorem and verify, it for

$$f(x) = x(x-1)(x-2)$$

in $\left[0, \frac{1}{2}\right]$. (7.5)

(c) Find the Maclaurin series expansion for the function

$$f(x) = \cos(2x).$$ (7.5)

4. (a) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent for $p > 1$ and divergent for $p \leq 1$. (7.5)

(b) State limit comparison test and hence, check the convergence of the series

$$\sum_{n=1}^{\infty} \frac{n+1}{n^2+n+1}.$$ (7.5)

5. (a) Define uniform convergence of the series of the function. Check the uniform convergence for the following series of the functions :

(i) $\sum_{k=1}^{\infty} \frac{x}{k^3}$ on $[-1,1]$.

(ii) $\sum_{k=1}^{\infty} \left(\frac{3x}{7}\right)^k$ on $[-2,2]$. (7.5)

(b) Show that the sequence of function $\{f_n\}$ defined by

$$f_n(x) = \frac{x}{1+nx^2},$$

converges uniformly on R. (7.5)

(c) Find the local maxima and local minima of the function

$$f(x) = 5x^3 + 2x^2 - 3x$$

on R. (7.5)