This question paper contains 8 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2424**

Unique Paper Code : **2362301** F-4

Name of the Paper : **Introduction to Operational Research and Linear Programming**

Name of the Course : **B.Tech. Computer Science/B.Sc. (Hons.) Statistics : Allied Course**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

This question paper has three sections in all.

Attempt any *Five* questions from each Section.

All questions carry equal marks.

All Sections are compulsory.

Use of simple calculator is allowed.

Section A

1. Discuss any *five* principles of modeling in operations research.

2. Define the term Basis of a vector space. Check whether the given set of vectors [3, 0, 2], [7, 0, 9], [4, 1, 2] form a basis of $\mathbf{R}^3$ or not.

3. Define basic solution of a Linear Programming Problem. Find all basic feasible solutions of the equations given by :

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$

$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2.$$

P.T.O.

4. Define a convex set. Examine the convexity of the following set :

$$X = \{(x_1, x_2) \mid x_1 x_2 \geq 1, x_1 \geq 0, x_2 \geq 0\}.$$

5. Ozark farms uses at least 800 lb of special feed daily. The special feed is a mixture of corn and soybean meal with the following compositions :

Feedstuff	lb per lb of feedstuff		
	Protein	Fiber	Cost ($/lb)
Corn	.09	.02	.03
Soybean meal	.60	.06	.90

The dietary requirements of the special feed are at least 30% protein and at most 5% fiber. Formulate a Linear Programming Problem for Ozark farm to determine the daily minimum-cost feed mix.

6. Consider the following LPP :

Maximize $z = 3x_1 + 2x_2$

Subject to :

$$x_1 + x_2 \leq 4$$

$$x_1 - x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

(a) Determine all the basic solutions of the problem and classify them as feasible and infeasible.

(b) Show how the infeasible basic solutions are represented on the graphical solution space.

Section B

7. Discuss the following special cases that arise in LPP :

(*a*) Degeneracy

(*b*) Infeasible solutions.

8. Use graphical method to solve the following LPP :

Maximize $z = 3x_1 + 2x_2$

Subject to :

$$5x_1 + x_2 \geq 10$$

$$2x_1 + 2x_2 \geq 12$$

$$x_1 + 4x_2 \geq 12$$

$$x_1, x_2 \geq 0$$

9. Consider the following system of equations :

$$x_1 + 2x_2 - 3x_3 + 5x_4 + x_5 = 4$$

$$5x_1 - 2x_2 + 6x_4 + x_6 = 8$$

$$2x_1 + 3x_2 - 2x_3 + 3x_4 + x_7 = 3$$

$$-x_1 + x_3 - 2x_4 + x_8 = 0$$

$$x_1, x_2, \ldots, x_8 \geq 0$$

Let x_5, x_6, x_7 and x_8 be a given initial basic feasible solution. If x_1 becomes basic, which of the given basic variables must become non-basic at zero level for all the variables to remain non-negative and what would be the value of x_1 in the new solution ? Repeat this procedure for x_2, x_3 and x_4.

10. Use Big M method to solve the following LPP :

Maximize $z = 3x_1 + 2x_2$

Subject to :

$$2x_1 + x_2 \leq 2$$

$$3x_1 + 4x_2 \geq 12$$

$$x_1, x_2 \geq 0$$

11. Describe alternate optimal solutions in LPP. Find any *three* alternate optimal solution (if they exist) for the following LPP :

Maximize $z = x_1 + 2x_2 + 3x_3$

Subject to :

$$x_1 + 2x_2 + 3x_3 \leq 10$$

$$x_1 + x_2 \leq 5$$

$$x_1 \leq 1$$

$$x_1, x_2, x_3 \geq 0.$$

12. Solve the following problem by inspection, and justify the method of solution in terms of the basic solutions of the simple method :

Maximize $z = 5x_1 - 6x_2 + 3x_3 - 5x_4 + 12x_5$

Subject to :

$$x_1 + 3x_2 + 5x_3 + 6x_4 + 3x_5 \leq 90$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0.$$

Section C

13. For the linear programming problem given by :

Max $z = 3x_1 + 2x_2 + 5x_3$

(Coefficients 3, 2, 5 represent the unit revenue for x_1, x_2 and x_3 respectively)

Subject to :

$$x_1 + 2x_2 + x_3 \leq 430 \text{ (Operation 1)}$$

$$3x_1 + 2x_3 \leq 460 \text{ (Operation 2)}$$

$$x_1 + 4x_2 \leq 420 \text{ (Operation 3)}$$

$$x_1, x_2, x_3 \geq 0.$$

The optimal table obtained from simple method is as follows :

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	4	0	0	1	2	0	1350
x_2	$\frac{-1}{4}$	1	0	$\frac{1}{2}$	$\frac{-1}{4}$	0	100
x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230
x_6	2	0	0	–2	1	1	20

where x_4, x_5 and x_6 are the slack variables for constaints of operations 1, 2 and 3 respectively.

Determine the range of unit revenue for x_1 for which the present solution remains optimal for the given problem.

14. Obtain the dual problem of the following primal problem :

Minimize $z = x_1 - 3x_2 - 2x_3$

Subject to :

$$3x_1 - x_2 + 2x_3 = 7$$

$$2x_1 + 2x_2 \geq 12$$

$$x_1 + 4x_2 \geq 12$$

$$x_1, x_2 \geq 0, x_3 \text{ is unrestricted.}$$

15. Consider the following linear programming problem :

Maximize $z = 2x_1 + 2x_2 + 4x_3$

Subject to :

$$2x_1 + x_2 + x_3 \leq 2$$

$$3x_1 + 4x_2 + 2x_3 \geq 8$$

$$x_1, x_2, x_3 \geq 0.$$

Show that Phase-1 will terminate with an artificial basic variable at zero level. Perform only one iteration at Phase-2.

16. Use dual Simplex Method to solve the given LPP :

Maximize $z = -x_1 - 3x_2$

Subject to :

$$x_1 - x_2 \leq 2$$

$$x_1 + x_2 \geq 4$$

$$2x_1 - 2x_2 \geq 3$$

$$x_1, x_2 \geq 0.$$

17. Consider the following LPP :

Maximize $z = 5x_1 + 2x_2 + 3x_3$

Subject to :

$$x_1 + 5x_2 + 2x_3 \leq b_1$$

$$x_1 - 5x_2 - 6x_3 \leq b_2$$

$$x_1, x_2, x_3 \geq 0$$

The following optimal tableau corresponds to specific values of b_1 and b_2.

Basic	x_1	x_2	x_3	x_4	x_5	**Solution**
z	0	a	7	d	e	150
x_1	1	b	2	1	0	30
x_5	0	c	−8	−1	1	10

Determine the following :

(*a*) The right-hand-side values b_1 and b_2

(*b*) The elements a, b, c, d, e.

18. Discuss economic interpretation of duality.