[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 2422 **F–4** **Your Roll No.....**

Unique Paper Code : 2352301

Name of the Course : **Applied Course : Mathematics / Chemistry / Geology**

Name of the Paper : Calculus

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Do any **five** questions from each of the three sections.
3. Each question is for **5** marks.

SECTION I

1. Given $f(x) = 3 - 2x$, $x_0 = 3$, $\epsilon = 0.02$. Find $L = \lim_{x \to x_0} f(x)$. Then find a number $\delta > 0$ such that for all x, $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \epsilon$.

2. (a) Find linearization of $f(x) = \text{Cos}(x)$ at $x = \pi/2$. (2.5)

   (b) Find $\lim_{\theta \to 0} \dfrac{\sin(\sqrt{2}\theta)}{\sqrt{2}\theta}$. (2.5)

3. Find asymptotes of the graph of $f(x) = \dfrac{x^2 - 3}{2x - 4}$.

4. Find the critical points, points of inflection and intervals of concavity up/down for $y = x^3 - 3x + 2$. Hence draw rough sketch.

5. Evaluate $\lim_{x \to 0} \dfrac{x(\text{Cos}(x) - 1)}{\text{Sin}(x) - x}$.

6. Find the volume of the solid generated when the region between the graphs of the equations $f(x) = \dfrac{1}{2} + x^2$ and $g(x) = x$ over the interval $[0,2]$ is revolved about x- axis.

P.T.O.

SECTION II

7. Evaluate $\int_1^4 \frac{dx}{(x-2)^{2/3}}$.

8. Find the arc length of the curve $y = x^{3/2}$, $0 \le x \le 1$.

9. Find area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$, and $x = 2$ about y - axis.

10. Sketch the curve $r = \text{Cos}(2\theta)$, $\theta = 0, \pm\frac{\pi}{2}, \pi$.

11. For what values of p does the integral $\int_1^\infty \frac{dx}{x^p}$ converge ? When the integral does converge what is its value ?

12. (a) Show that $r(t) = (\text{Cos}(t))i + \sqrt{5}j + \text{Sin}(t)k$ has constant length and is orthogonal to its derivative.

    (b) Find the unit tangent vector of the curve $r(t) = 6t^3i - 2t^3j - 3t^3k$, $1 \le t \le 2$.

SECTION III

13. Find the curvature for the curve $r(t) = 2\text{Cos}(t)i + 3\text{Sin}(t)j$, $0 \le t \le 2\pi$.

14. Find the plane tangent to a surface $z = 1 - \frac{1}{10}(x^2 + 4y^2)$ at $\left(1, 1, \frac{1}{2}\right)$.

15. Find the local maxima, local minima and saddle point of the function

$$f(x, y) = x^3 - y^3 - 2xy + 6$$

16. (a) Find $\lim_{(x,y)\to(1,1)} \frac{xy - y - 2x + 2}{x - 1}$.

    (b) Find $\frac{\partial w}{\partial v}$ when $u = 0$, $v = 0$ if $w = x^2 + \left(\frac{y}{x}\right)$, $x = u - 2v + 1$, $y = 2u + v - 2$.

17. Find the derivative of $f(x,y,z) = xy + yz + zx$ at $P_0(1, -1, 2)$ in the direction of $3i + 6j - 2k$.

18. Find gradient of the function at the given point

    (a) $f(x,y) = \ln(x^2 + y^2)$; $(1,1)$

    (b) $g(x,y) = y - x^2$; $(-1,0)$