This question paper contains **7** printed pages]

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory.

Attempt any *four* questions from the rest.

Scientific calculators (non-programmable) are allowed.

Begin answering each question on fresh page of the answer-sheet.

1. Attempt any *five* questions :   5×3=15

    (*a*) Define truncation error and round off error.

    (*b*) What are transcendental and algebraic equations ? Give *one* example of each.

    (*c*) Give geometrical interpretation of trapezoidal rule.

    (*d*) Give geometrical interpretation of Newton-Raphson method.

(e)   Find the minimum number of iterations required to attain an accuracy of 0.001 in an interval [1, 2] using bisection method.

(f)   Distinguish between Newton-Raphson method and Secant method giving at least *three* differences between them.

(g)   Find the eigen values and normalized eigen vectors of the matrix :

$$\begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix}.$$

2.   (a)   Using Gauss-Seidel method and the first iteration as (0, 0, 0), calculate the next three iterations for the solution of the system of equations :

$$5x - y + z = 10$$

$$2x + 8y - z = 11$$

$$-x + y + 4z = 3.$$

(b)   Find the largest (dominant) eigen value of the matrix :

$$\begin{pmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{pmatrix}.$$

(c) Find a real root of the equation :

$$x^3 - 5x - 7 = 0$$

using secant method. (Three interation). 3×5=15

3. (a) Construct the dividend difference table for the data :

X	F(x)
0.5	1.625
1.5	5.875
3.0	31.0
5.0	131.0
6.5	282.125
8.0	521.0

Hence find the interpolating polynomial and an approximation to the value of F(7).

(b) The following table contains data representing a polynomial of degree 5. In the table $\overset{\smile}{y}$ at $x = 3$ is in error. Correct the error :

x	y
0	1
1	2
2	33
3	254
4	1054
5	3126
6	7777

(c) Find the first and second derivative of $f(x)$ at $x = 1.5$ from the following tabulated values :

x	f(x)
1	3.23

1.5   3.19

2   3.0

2.5   2.91

3   2.81

Compute the result upto two places of decimal.   3×5

4. (a) Evaluate :

$$\int_0^1 \frac{dx}{1 + x^2}$$

using Simpson's rule for $n = 4$, correct to five decimal places.

(b) Evaluate the value of integral using Gauss-Legendre's three point formula ($n = 2$) correct

upto five decimal places for the following :

$$\int_{-1}^1 e^x \, dx$$

(c) Derive second order Runge-Kutta method.   3×5

5. (a) Using Euler's method find the approximate value of y when $x = 0.3$:

$$\frac{dy}{dx} = x + y^2;$$

$$y(0) = 1 \text{ and } h = 0.1.$$

(b) Fit a quadratic function to the following data points :

x	$y(x)$
1	1
2	16
3	81
4	256

(c) The distance covered by a car in a given time is tabulated below :

Time (mins)	Distance Covered (Km)
10	12
12	15

14	20
16	27
18	37

Find the speed and acceleration of the car at time = 10 min.   3×5

6. (a) What is truncated approximate value of $\frac{\pi}{4}$ upto four significant digits ? Calculate the absolute and relative errors due to truncation and due to systematic rounding. Take actual value of π as $\frac{22}{7}$ and approximate value of $\frac{\pi}{4}$ as 0.7854.

(b) Find the roots of the equation :

$$x \sin x + \cos x = 0$$

to three decimal places by Newton-Raphson method.

(c) Describe Gram-Schmidt orthogonalization process.   3×5