This question paper contains **4** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7846**

Unique Paper Code : **2351101** **F-1**

Name of the Paper : **Calculus-I [DC-1.1]**

Name of the Course : **B.Sc. (Hons.)**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Five* questions from each Section.

All questions carry equal marks.

Section I

1. If

$$y = x \log\left(\frac{x-1}{x+1}\right),$$

prove that :

$$\frac{d^n y}{dx^n} = (-1)^n (n-2)! \left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right].$$

2. Find constants a and b that guarantee that the graph of the function defined by :

$$f(x) = \frac{ax+5}{3-bx}$$

will have a vertical asymptote at $x = 5$ and a horizontal asymptote at $y = -3$.

P.T.O.

3. Find :

$$\lim_{x \to 0} \left(e^x - 1 - x\right)^x.$$

4. Suppose a manufacturer estimates that, when the market price of a certain product is p, the number of units sold will be :

$$x = -6 \ln\left(\frac{p}{40}\right).$$

It is also estimated that the cost of producing these x units will be :

$$C(x) = 4xe^{\frac{-x}{6}} + 30.$$

(*a*) Find the average cost, the marginal cost, and the marginal revenue for this production process.

(*b*) What level of production x corresponds to maximum profit ?

5. Compute :

$$\int_0^{\pi/3} \sin^4 3x \cos^3 3x \, dx.$$

6. Trace the polar curve $r = \cos 2\theta$.

Or

Find the interval of increase and decrease and of concavity for the function :

$$f(x) = 4 + \frac{2x}{x-3}.$$

Also find the vertical and horizontal asymptotes of this function, if any.

Section II

7. Derive the formula for the volume of a right pyramid whose altitude is h and whose base is a square with sides of length a.

8. Use the method of cylindrical shells to find the volume of the solid generated when the region R enclosed by the curves :

$$xy = 4 \text{ and } x + y = 5$$

is revolved about the x-axis.

9. Find the area of surface generated by revolving the curve C :

$$x = 2\sqrt{1-y}, -1 \leq y \leq 0$$

about the y-axis.

10. Find the equation of the ellipse that has its foci at (2, 1) and (2, –3) and which has the major axis of length 6.

11. By rotating the coordinate axes remove the xy-term from the equation $xy = -9$. Hence or otherwise name the conic represented by the equation.

12. State the reflection property of Hyperbola. Find the equation of the hyperbola whose vertices are at (2, 4) and (10, 4) and the foci 10 units apart.

Section III

13. Find the arc length of the parametric curve, without eliminating the parameter, given by :

$$x = \cos t + t \sin t,$$

$$y = \sin t - t \cos t,$$

$$0 \leq t \leq \pi.$$

14. Test if the vector function $\mathbf{F}(t) = (t^2 - t, \cos t, \sin t^2)$ over $[-\pi, \pi]$ is smooth.

15. (*i*) Show that if the non-zero vector function $\mathbf{F}(t)$ is differentiable and has constant length, then $\mathbf{F}(t)$ is orthogonal to the derivative vector $\mathbf{F}'(t)$.

(*ii*) If

$$\mathbf{G} = \mathbf{F}.(\mathbf{F}' \times \mathbf{F}''),$$

what is $\mathbf{G}'$?

16. At what angle (to the nearest tenth of a degree) should a projectile be fired from ground level if its muzzle speed is 167.1 ft/s and the desired range is 600 ft ?

17. Find the curvature and the radius of curvature of the vector function :

$$r(t) = 3\cos ti + 4 \sin tj + tk$$

at the point where $t = \pi/2$.

18. At a certain instant, the velocity and acceleration of an object are given by (5, –1, 2) and (1, 0, –7). Find the tangential and normal component of the velocity and acceleration at this instant.