This question paper contains 6 printed pages.]

Your Roll No. .....

1368 **A**

B.Sc. (Hons.)/I

BIOCHEMISTRY—Paper IV

(Mathematics and Statistics)

(Admissions of 2000 and onwards)

Time : 3 *Hours* *Maximum Marks* : 60

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt ***five*** *questions in all, selecting at least* ***two*** *questions from each Section.*

SECTION–A

1. (a) If $f(x) = \log \frac{\sqrt{a+bx} - \sqrt{a-bx}}{\sqrt{a+bx} + \sqrt{a-bx}}$, find for what value of x, $\frac{1}{f'(x)} = 0$. 4

(b) Let $f(x) = \frac{1}{x}$, $x \in (0,1]$. Examine the validity of the hypothesis and conclusion of Lagrange's Mean Value Theorem for the above function. 4

(c) If $y = \sin(m \sin^{-1} x)$, show that $(1-x^2)y_{n+2} = (2n+1)xy_{n+1} + (n^2 - m^2)y_n$. 4

[P.T.O.

2. (a) Show that

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + (-1)^n \frac{x^{2n-1}}{(2n-1)!} + \ldots \forall\, x \in \mathbb{R}. \quad 4$$

(b) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. 4

(c) Prove that for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the radius of curvature $\rho = \frac{a^2b^2}{p^3}$, where p is the perpendicular distance from its center upon the tangent at (x, y). 4

3. (a) Find all the asymptotes of the curve :

$$3x^3 + 2x^2y - 7xy^2 + 2y^3 - 14xy + 7y^2 + 4x + 5y = 0 \quad 4$$

(b) Find the maxima and minima of the function f defined by,

$$f(x) = 2x^3 - 15x^2 + 36x + 10 \text{ for } x \in \mathbb{R}. \quad 4$$

(c) Evaluate $\lim_{x \to \pi/2}\left(\sec x - \frac{1}{1-\sin x}\right)$. 4

4. (a) Trace the curve :

$$y(1 - x^2) = x^3. \quad 4$$

(b) Find the equation of the normal to the curve :

$$y(x-2)(x-3) - x + 7 = 0$$

at the point where it cuts the axis of X. 4

(c) Find the position and nature of multiple points of the following curve :

$$x^3 + y^3 + a^3 = 3axy. \quad 4$$

5. (a) Solve the equation :

$$27x^3 + 42x^2 - 28x - 8 = 0$$

the roots being in geometric progression. 4

(b) If $x_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$, find the value of x_1, x_2, x_3 ..... to ∞. 4

(c) Find the rank of the matrix :

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & -2 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & 0 & 3 \end{pmatrix}$$ 4

6. (a) If $I_n = \int_0^{\pi/2} x\sin^n x dx$ and $n > 1$, prove that

$$I_n = \frac{n-1}{n} I_{n-2} + \frac{1}{n^2}.$$ 4

(b) Solve the following differential equation :

$$xdy - ydx - b\sqrt{x^2 - y^2}dx = 0.$$ 4

(c) Examine the convergence of the following sum :

$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \ldots$$ 4

SECTION–B

7. (a) If $\bar{X}_\omega$ is the weighted mean of X_i's with weights ω_i then prove that,

$$\left(\sum_{i=1}^{n}\omega_i\right)\left(\sum_{i=1}^{n}\omega_i(X_i - \bar{X}_\omega)^2\right) = \sum_{i=1}^{n}\sum_{j>i}\omega_i\omega_j(X_i - X_j)^2$$

where $\sum_{i=1}^{n}\omega_i \neq 0$. 4

[P.T.O.

(b) From a sample of n observations the arithmetic mean and variance are calculated. It is then found that one of the values x_1 is in error and should be replaced by x'_1. Show that the adjustment to the variance to correct this error is

$$\frac{1}{n}(x'_1 - x_1)\left\{x'_1 + x_1 - \frac{x'_1 - x_1 + 2T}{n}\right\}$$

where T is the total of the original observations. 4

(c) A and B throw a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins show that his chance of winning is $\frac{30}{61}$. 4

8. (a) A person draws 2 balls from a bag containing 3 white and 4 red balls. If he is to receive Rs. 10 for every white ball drawn and Rs. 20 for each red ball. Find his expectation. 4

(b) A car hire farm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which neither car is used and the proportion of days on which some demand is refused. Given $(e^{1.5} = 0.2231)$. 4

(c) In a certain examination 2000 students appeared. The average marks obtained were 50% with standard deviation was 5%. If the marks are normally distributed, how many students do you expect to obtain more than 60% marks? (Area under the standard normal curve from $x = 0$ to $x = 2$ is 0.4772.) 4

9. (a) The following table gives the joint probability distribution of (X,Y):

X / Y	1	2	3
1	$\frac{5}{27}$	$\frac{4}{27}$	$\frac{2}{27}$
2	$\frac{1}{27}$	$\frac{3}{27}$	$\frac{3}{27}$
3	$\frac{3}{27}$	$\frac{4}{27}$	$\frac{2}{27}$

Find:

(i) Marginal distribution of X and Y.

(ii) $P[1 \le X \le 2, Y \ge 2]$

(iii) $V(X), V(Y)$.

(iv) Cov (X, Y).

Symbols carry usual meaning. $(1+1+1+1=4)$

(b) Let X_1, X_2 be two random variables with zero means and variances σ_1^2 and σ_2^2 respectively having correlation coefficient r between them. Determine the values of the constants a and b which are independent of r such that the random variables $X_1 + aX_2$ and $X_1 + bX_2$ are uncorrelated. 4

(c) The lines of regression obtained in a certain correlation analysis are:

$X + 9Y = 7$ & $Y + 4X = 16\frac{1}{3}$

Find

(i) Correlation coefficient of X and Y

(ii) σ_X, σ_Y 4

10. (a) In a large city A, 20% of random sample of 900 school boys had a certain physical defect. In another large city B, 18.5% of a random sample of 1600 school boys had same defect. Is the difference between the proportions significant? 4

(b) In an experiment on immunization of cattle from tuberculosis the following results were obtained :

	Affected	Unaffected
Inocculated	17	23
Not Inocculated	8	12

Examine the effect of vaccine in controlling the incidence of the disease.

[Given $\chi^2_{0.05} = 3.84$ for 1 degree of freedom.] 4

(c) For a random sample of 10 pigs fed on a diet A the increase in weight in pounds in a certain period were, 10, 06, 16, 17, 13, 12, 08, 14, 15, 09 lbs. For another sample of 12 pigs fed on diet B the increase in the same period were 07, 13, 32, 15, 12, 14, 18, 08, 21, 23, 10, 17 lbs. Test whether diets A and B differ significantly as regard the effect on the increase in weights. You may use the fact that 5% value of t for 20 d.f. is 2.09. 4