



- (c) Examine the continuity of the following function at  $x = \frac{1}{2}$

$$f(x) = \begin{cases} \frac{1}{2} - x & 0 < x < \frac{1}{2} \\ 0 & x = \frac{1}{2} \\ \frac{3}{2} - 3x & \frac{1}{2} < x < 1 \end{cases} \quad (5\frac{1}{2})$$

3. (a) Consider a unicellular organism of cylindrical shape having radius 'r' and height 'h'. Show that if the height remains constant while the radius changes, the rate of change of volume V with respect to the radius will be  $2V/r$ . (5)

- (b) If  $y = a \cos(\log x) + b \sin(\log x)$ , prove that  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ . (5½)

- (c) Evaluate the following integrals

(i)  $\int xe^x dx$

(ii)  $\int_0^1 \frac{x}{x^2+1} dx$  (6)

## SECTION II

4. (a) Let the matrix  $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix}$  and  $B = \begin{bmatrix} -1 & -2 \\ 0 & 5 \\ 3 & 1 \end{bmatrix}$ . Find the matrix X such that  $A + B = 2X + O$ . (5)

- (b) Find the values of x such that  $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 5 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$ . (6)

5. (a) Find the image of the point  $(-2, 2)$  under (i) reflection in origin and (ii) contraction by a scale factor  $\frac{1}{2}$  by using matrix multiplication. (6)

(b) Find  $\lambda$  and  $\mu$  if  $\begin{bmatrix} 2 & -3 \\ \lambda & \mu \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$ . (5)

6. (a) Given  $f(x) = x^2 - 5x + 6$ , and  $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ , find  $f(A)$ . (5)

(b) For the matrix  $A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{-2}{3} \\ \frac{-2}{3} & \frac{2}{3} & \frac{-1}{3} \end{bmatrix}$ , find its transpose. Also verify that  $AA^T = I$ . (6)

### SECTION III

7. (a) The following table gives the wages of 120 workers in an industry. Compute their median wage.

Wages in Rs.	20	21	22	23	24	25	26	27	28
No. of workers	8	10	11	16	20	25	15	9	6

 (5)

- (b) Find the standard deviation and the coefficient of variation of the following data :

Age (in years)	5-7	7-9	9-11	11-13	13-15
Frequency	16	13	10	6	5

 (5)

8. (a) A medical research team wished to evaluate a screening test for Alzheimer's disease. The test was given to 450 patients with symptoms and 500 without the symptoms.

Test Symptoms	Yes (D)	No (D <sup>c</sup> )	Total
Positive (T)	436	5	441
Negative (T <sup>c</sup> )	14	495	509
Total	450	500	950

Find (i)  $P(D \cap T)$ , (ii)  $P(D)$ , (iii)  $P(T)$ , (iv)  $P(D/T)$  and (v)  $P(T/D)$  (5)

- (b) The incidence of an occupational disease in an industry is such that the workers have a 20% chance of suffering from it. What is the probability that out of six workers chosen at random, four or more will suffer from the disease? (5)

9. (a) For a bivariate distribution, the lines of regression are  $3x + 12y = 19$  and  $3y + 9x = 46$ . Find the mean and the coefficient of correlation. (5)

- (b) Find the line of best fit to the following data by treating  $y$  as the dependent variable and  $x$  as the independent variable.

x	6	7	7	8	8	8	9	9	10
y	5	5	4	5	4	3	4	3	3

(5)

10. (a) A random sample of 20 items has mean 42 and standard deviation 5 units. Test the hypothesis that the random sample is drawn from a normal population with mean 45. (Given that  $t_{0.05, 19} = 2.09$ ) (5)

- (b) Assume the mean heights of soldiers to be 68.22 inches with a variance of 10.8 square inches. How many soldiers in a regiment of 1000 would you expect to be more than 6 feet tall? (5)