[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1034 **E** **Your Roll No.....**

Unique Paper Code : 235161

Name of the Course : **B.Sc. (H) / Bio Chem. / Bio Med. Sc. / Microbiology**

Name of the Paper : MATHEMATICS AND STATISTICS [MACT 303]

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. There are **three** sections in this paper.
3. Attempt any **two** questions from each Section.
4. Use of simple calculators, log tables and normal tables are allowed.

SECTION – I

1. (a) Out of 450 students in a college, 193 know German language and 200 know French, 80 know neither. Illustrated the fact by Venn diagram and find out how many know both.

   (b) A, B and C are sets of real numbers defined as :
   $A = \{x : 2 < x \leq 4\}$, $B = \{x : x^2 - 4x + 4 > 0\}$ $C = \{x : 3 - x < x - 1\}$
   Determined which of A, B and C are subsets of one another.

   (c) A diet is contained 400 units of carbohydrate, 500 units of fat. A unit of F_1 contains 10 units of carbohydrate, 20 units of fat, a unit of F_2 contains 25 units of carbohydrate, 10 units of fat. Find the possible combinations of two foods in a diet to meet the minimum nutrition requirement. Depict the results graphically also. (5,5,5)

2. (a) Determine the value of h if $f(x) = \begin{cases} x^2 + h & \text{if } x \neq 1 \\ 3 & \text{if } x = 1 \end{cases}$ is continuous at x = 1.

(b) Integrate :

(i) $\int x^2 e^{x^3} dx$

(ii) $\int \frac{x^2}{(x^3+1)^2} dx$

(c) In a chemical reaction temperature T varies with the time t accordingly to the formula

$$T = \frac{3t+1}{t+2}$$

Find the rate of change of T with respect to t. (5,5,5)

3. (a) Let $f(x) = x^2 + 3x + 1$, find

(i) The average rate of change of f(x) with respect to x over the interval [3,6].

(ii) The derivative of f(x) at x = 4.

(b) If $y = e^{-x} \cos x$, prove that $\frac{d^2y}{dx^2} = 2\, e^{-x} \sin x$.

(c) (i) Write the first five terms of the sequence by recursion formula

$$a_1 = 2, \quad a_{n+1} = (-1)^{n+1} \frac{a_n}{2}.$$

(ii) Find the sum of infinite series $3 + \frac{6}{2} + \frac{12}{25} + \frac{24}{125} + \ldots\ldots\ldots$. (5,5,5)

4. (a) Evaluate :

(i) $\lim_{x \to 0} \frac{\tan 3x}{\sin 5x}$

(ii) $\lim_{x \to \infty} \frac{6 - x^3}{7x^3 + 3}$

(b) In a certain culture of bacteria, the rate of increase of bacteria is proportional to the number present. It is found that there are 1000 bacteria at the end of 3 hours and 4000 at the end of 5 hours, how many were there in the beginning.

(c) Find the interval in which the function $f(x) = -x^2 + 9x - 20$ is increases or decreases. (5,5,5)

SECTION – II

5. (a) If $A = \begin{bmatrix} 2 & -1 \\ 4 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 3 & 5 \end{bmatrix}$, is $A^2 - B^2 = (A+B)(A-B)$?

(b) Find the image of point (1,–3) under the following transformations using matrix multiplication :

(i) dilation by a factor 5.

(ii) rotation through an angle 30° in counter clockwise direction.

(iii) reflection in y-axis. (4,6)

6. (a) If $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 \\ 0 & 3 \\ 5 & 1 \end{bmatrix}$, find matrix X such that $A + B - 2X = 0$.

(b) Find the values of x, y, z and w which satisfies the matrix equation

$$\begin{bmatrix} x+y & x-2z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} 3 & -9 \\ 0 & 18 \end{bmatrix} \quad (4,6)$$

7. (a) If $f(x) = x^2 - 4x + 6$ and $A = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$ find f(A).

(b) If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$, find a and b such that AB = BA.

(4,6)

SECTION – III

8. (a) The mean marks obtained by 300 students in statistics is 45. The mean of top 100 of them was found to be 70 and mean of 100 was known to be 20. What is the mean of remaining students ?

(b) The sum of 50 observations is 500, its sum of squares is 6000 and median is 12. Find the coefficient of variation and coefficient of skewness. (6,6½)

9. (a) Obtained the equation of straight line that is best fit to the data :

X:	1	2	3	4	5
Y:	5	7	9	10	11

Give an estimate of Y for X = 6.

(b) The two lines of regression are given by $8x + 10y = 25$ and $16x + 5y = 12$. If the variance of x is 25, what is the standard deviation of y ? (6,6½)

10. (a) Assuming the birth of boys and girls to be equally likely, what proportion of families will have

(i) exactly two boys and two girls

(ii) at least two girls.

(b) The height of the plants of a certain species are normally distributed with mean height being 30 cm and standard deviation being 5 cm. What proportions of plants are between 40 cm and 50 cm in height. (6,6½)

11. (a) The means of two single large samples of size 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the sample be regarded as drawn from the population of standard deviation of 2.5 inches ? (Test at 5% level of significance).

(b) A random variable X has the following probability function :

X	:	0	1	2	3	4	5	6	7
P(X)	:	0	2k	3k	k	2k	k^2	$7k^2$	$2k^2+4$

Find the value of k and the variance σ_x^2. (6,6½)