This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8442**

Unique Paper Code : **235164** **C**

Name of the Paper : **MACT-101 Mathematics-I**

Name of the Course : **B.Sc. (Hons.) Chemistry Part I**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are 3 sections in this question paper. Attempt any *two* questions from each section. Students are allowed to use scientific calculator without programming.

Section I

1. (*a*) An actual volume of 25.00 cm^3 is measured as 25.15 cm^3. Calculate the following :

   (*i*) The absolute uncertainty.

   (*ii*) The fractional uncertainty.

   (*iii*) The percentage uncertainty. 5

   (*b*) Draw the graph of : 5

$$y = \frac{1}{2} e^x - 3.$$

2. (*a*) Use Trapezoidal rule to evaluate $\int_1^5 \frac{dx}{x}$ by dividing 1 to 5 into 8 equal parts. 5

(*b*) (*i*) For which value of K, the equation $x^2 - 5x + k = 0$ has real and unequal roots. If $k = 2$, then find the solution of the equation.

(*ii*) For which value of *m* will the equation $m^2t^2 + 2(m + 1)t + 4 = 0$, have exactly one zero. 5

3. (*a*) Find the root of the equation $x^3 - 5x + 3 = 0$, between 0.5 and 0.75 up to 3 decimals by Newton-Raphson method. 5

(*b*) Evaluate : 5

$$\lim_{x\to 0} \frac{\tan x - x}{x^2 \tan x}.$$

Section II

4. (*a*) Evaluate : 6

$$\int \frac{1}{(x + 1)^2 (x^2 + 1)} dx.$$

(*b*) Find the local extrema and inflexion points of

$$f(x) = 2x^3 - 3x^2 - 12x + 3$$

over the entire *x*-axis. 6½

5. (*a*) (*i*) Evaluate : 4

$$\lim_{x\to 0} (\cot x)^{1/\log x}.$$

(*ii*) Two intervals have been clocked as $56.15s \pm 0.13s$ and $75.12s \pm 0.17s$. Find the probable value of their sum and its probable error. 2

(*b*) Find the Maclaurin series and interval of convergence for :

$$f(x) = \frac{1}{x - 1}.$$ 6½

6. (*a*) Examine the continuity of the function : 6

$$f(x) = |x| + [x]$$

at $x = 1, -1/2$.

(*b*) Evaluate : 6½

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx.$$

Section III

7. (*a*) If $C(x, t) = (4\pi Dt)^{-1/2}\, e^{-x^2/4Dt}$, then prove that : 5

$$\frac{\partial C}{\partial T} = D\frac{\partial^2 C}{\partial x^2}.$$

(*b*) If 5

$$f(x, y) = ae^{-b(x^2 - y^2)},$$

then evaluate $\left(\frac{\partial^2 f}{\partial x^2}\right)_y$ and $\left(\frac{\partial^2 f}{\partial y^2}\right)_x$.

(*c*) If $x = \sin t$, $y = \sin(pt)$, then show that : 5

$$(1 - x^2)\frac{d^2 y}{dx^2} - x\frac{dy}{dx} + p^2 y = 0.$$

8. (*a*) For an ideal gas equation

$$PV = nRT,$$

where P is the pressure, V is the volume, R is an ideal gas constant, n is the number of moles and T is the temperature. Find an expression for dP and calculate the change in pressure of an ideal gas, if the volume is changed from 20.0001 to 19.8001, the temperature is changed from 298.15 K to 299.00 K, and the amount of gas in moles is changed from 1.0000 mol to 1.0015 mol. 5

(*b*) If 5

$$z = \log\left(\frac{x^3 + y^3}{x + y}\right).$$

then show that :

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2.$$

(*c*) Find the n^{th} derivative of : 5

$$y = \sin^4 x.$$

9. (*a*) Perform the line integral

$$\int_C du = \int_C x^2 y dx + xy^2 dy$$

wherec C represents the line segment from (0, 0) to (2, 2). Also perform the line integral from (0, 0) to (2, 0) and then from (2, 0) to (2, 2). 5

(*b*) (*i*) Test the convergence of the series

$$\sum_{n=0}^{\infty} \frac{1}{3^n + x}, x > 0.$$

(*ii*) Show that $\ln(y) = (2.302585....)\log_{10} y$.

(*iii*) Find the value of $\log_{10} 0.01$. 5

(*c*) If $x = 2 \cos t - \cos 2t$, $y = 2 \sin t - \sin 2t$,

then find the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$. 5