This question paper contains 7 printed pages.]

Your Roll No. .....

1255

B.Sc. (Hons.) / II A

CHEMISTRY – Paper X

(Mathematics - II)

Time : 3 Hours **Maximum Marks : 55**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **six** questions in all,
selecting **three** questions from each section.
Attempt each section on separate answer sheets.

Section – A

1. (a) Prove that every square matrix can be expressed uniquely as a sum of Hermitian and a skew-Hermitian matrices. 3

(b) Find the rank of the matrix

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -3 & 1 & -4 \\ 7 & -3 & -9 \\ 4 & -2 & 5 \end{pmatrix}$$ 3

(c) Find the characteristic equation of the matrix

$$A = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 3 \end{pmatrix}$$

and hence compute its inverse. **3**

2. (a) Define the following :

(i) Divergent sequence

(ii) Oscillatory sequence

(iii) Bounded sequence

Give one example of each of these. **3, 1½**

(b) Show that $\lim_{n\to\infty} n^{1/n} = 1$ **4½**

3. (a) State Cauchy's n^{th} root test for the convergence and divergence of a +ve term series. **2**

(b) Test for convergence the following series :

(i) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n-1}}{n}$ 3

(ii) $\frac{\log 2}{2^2} - \frac{\log 3}{3^2} + \frac{\log 4}{4^2} - \ldots$ 4

4. (a) Show that

$$2B(m, n) = \int_0^{\infty} \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}\, dx$$ 4

(b) Evaluate

(i) $\int_0^{\pi/2} \sin x\, dx \times \int_0^{\pi/2} \sin x \sqrt{\sin x}\, .dx$ 3

(ii) $\int_0^{\infty} e^{-x^2}\, dx.$ 2

5. (a) When the region of integration A is the triangle bounded by $y = 0$, $y = x$ and $x = 1$, show that

$$\iint_A \sqrt{4x^2 - y^2}\, dxdy = \frac{1}{3}\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)$$ 4½

(b) Show that

$$\int\int\int \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}} = \frac{\pi^2}{8},$$

the integral being extended to all the + ve values of the variables for which the expression $\sqrt{1-x^2-y^2-z^2}$ is real. 4½

Section – B

6. (a) Prove that for n arbitrary independent events $A_1, A_2, \ldots A_n$

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) + P(\bar{A}_1)\, P(\bar{A}_2) \ldots P(\bar{A}_n) = 1$$

and hence with $P(A_i) = 1 - \frac{1}{\alpha^i}$, $i = 1, 2 \ldots n$,

find the value of $P(A_1 \cup A_2 \cup \ldots \cup A_n)$. 5

(b) A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card. 4

7. (a) A discrete variate X assumes three values $-3, 0, 4$ and $P(X = 0) = \frac{1}{2}$. If $E(X) = \frac{9}{8}$, find $P(X = -3)$ and $P(X = 4)$. 3

(b) Show that for the binomial distribution with parameters n and p,

$$\mu_{r+1} = pq\left(nr\,\mu_{r-1} + \frac{d\mu_r}{dp}\right)$$ 4

(c) In a poisson frequency distribution, frequency corresponding to 3 successes is $\frac{2}{3}$ times frequency corresponding to 4 successes. Find the mean and standard deviation of the distribution. 2

8. (a) Show that for a normal distribution with mean μ and standard deviation σ, even order moments about mean are given by

$$\mu_{2n} = \sigma^2 (2n-1)\,\mu_{2n-2}.$$ 5

(b) Fit a straight line to the data taking x as the dependent variable : 4

x	y
1	1
3	2
4	4
6	4
8	5
9	7
11	8
14	9

9. (a) In a distribution exactly normal, 10.03% of the items are under 25 kilogram weight and 89.97% of the items are under 70 kilogram weight. What are the mean and standard deviation of the distribution ? 5

(b) If $4x - 5y + 33 = 0$

and $20x - 9y - 107 = 0$

are the two lines of regression, find

(i) the mean values of x and y.

(ii) the correlation coefficient. 4

10. (a) A die is thrown 264 times with the following results :

No. appeared on the die :	1	2	3	4	5	6
Frequency :	40	32	28	58	54	60

Show that the die is biased. **4½**

(b) A certain stimulus administered to each of the 12 patients resulted in the following increase of blood pressure :

5, 2, 8, –1, 3, 0, –2, 1, 5, 0, 4 and 6.

Can it be concluded that the stimulus will, in general, be accompanied by an increase in blood pressure ? **4½**

[Given that if

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t \exp(-x^2/2)dx,$$

then $f(1.28) = 0.3997$

$f(1.43) = 0.4236$

χ^2 for 5 d.f. at 5% level = 11.07

$t_{0.05}(11) = 2.201$].
