This question paper contains 4+2 printed pages]

Your Roll No. ....

5772

B.Sc. (Hons.) CHEMISTRY/III Sem. B

Paper MACT-302 : Mathematics—I

(Admission of 2010 and onwards)

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

This question paper has *three* Sections, *two* questions in each Section i.e. *six* questions in all. Attempt *two* parts from each question. *All* questions are compulsory.

Use of scientific calculator is allowed.

Section I

1. (*a*) Determine the solution of

$$y''(x) + y'(x) - 2y(x) = 0$$

subject to the conditions $y(0) = 0$ and $y'(0) = 6$. 6½

P.T.O.

(*b*) Find the solution by method of separation of variables :

$$y\frac{\partial f}{\partial x} - x\frac{\partial f}{\partial y} = 0.$$ 6½

(*c*) Find the value of the constant so that the following integral equals unity :

$$A \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2 - y^2} \, dy \, dx.$$ 6½

2. (*a*) Evaluate the double integral :

$$\int_0^a \int_0^b \left(x^2 + 4xy\right) dy \, dx.$$ 5

(*b*) Find the gradient of the function :

$$f = x^2 + 3xy + z^2 \sin\left(\frac{x}{y}\right).$$ 5

(*c*) Find $\vec{\nabla} \times \vec{F}$ if $F = \hat{i}y + \hat{j}z + \hat{k}x$. 5

Section II

3. (*a*) Find the stationary points of

$$f(x, y) = x^3 + y^3 - x - 6y + 10$$

and investigate the behaviour of them. 6½

(*b*) (*i*) Determine the curve in the complex plane that is described by $|z - 1| = 2$. 2½

(*ii*) Express $z_1 = 1 + i$ and $z_2 = -1 - i$ in terms of Euler's formula. 4

(*c*) Show that e^{ikx} is an eigen function of $\hat{P}_x$ where :

$$\hat{P}_x = -ih\frac{d}{dx}.$$

What is the corresponding eigenvalue ? 6½

4. (*a*) Show that :

$$\text{grad}\left(\frac{1}{|\vec{r}|}\right) = \frac{-\vec{r}}{|\vec{r}|^3},$$

where

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}.$$ 6½

(*b*) Show that the vectors :

$$v_1 = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k},$$

$$v_2 = \frac{1}{\sqrt{6}}\hat{i} - \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$$

and $$\vec{v}_3 = -\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$$

are of unit length and are mutually perpendicular. 6½

(*c*) Define orthogonal matrix and show that :

$$A = \frac{1}{9}\begin{pmatrix} 1 & 8 & -4 \\ 4 & -4 & -7 \\ 8 & 1 & 4 \end{pmatrix}$$

is an orthogonal mitrix. 2+4½

Section III

5. (*a*) Given :

$$\hat{A} = \frac{d}{dx} \text{ and } \hat{B} = x^2 \text{ (multiply by } x^2\text{)}$$

show that :

$$\hat{A}^2 f(x) \neq \left[\hat{A} f(x)\right]^2$$

and that

$$\hat{A}\hat{B} f(x) \neq \hat{B}\hat{A} f(x)$$

for arbitrary function $f(x)$. 3+3½

(*b*) Find the eigenvalues and eigenvectors of :

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$ 6½

(*c*) Solve the following set of equations by using the Cramer's rule :

$$x + y = 2$$

$$3x - 5y = 5.$$ 6½

6. (*a*) Solve the equation :

$$y''(x) + 3xy'(x) + 3y(x) = 0$$

by the power series method. 6½

(*b*) Find the inverse of the matrix :

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 2 & 0 \end{pmatrix}.$$ 6½

(*c*) Sketch the region of integration and evaluate the double integral :

$$\int_0^2 \int_x^{2x} \left(x^2 + y^2\right) dy\, dx.$$ 6½