[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 734 **G** **Your Roll No...............**

Unique Paper Code : 235365

Name of the Paper : Mathematics – II (MACT-302)

Name of the Course : **B.Sc. (Hons.) Chemistry – II**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt **two** parts from each question.
4. **All** questions are compulsory.
5. Use of scientific calculator is allowed.

1. (a) Solve the initial value problem $y''(t)+9y(t)=0,\ y\left(\frac{\pi}{3}\right)=0,\ y'\left(\frac{\pi}{3}\right)=-1$. (6½)

   (b) Solve the partial differential equation by Method of Separation of variables $y\frac{\partial f}{\partial x}-x\frac{\partial f}{\partial y}=0$. (6½)

   (c) Use the power-series method to solve the differential equation $y''(x)-y(x)=0$ and show that the solution can be expressed in the form $c_1e^{x}+c_2e^{-x}$. (6½)

2. (a) Transform to polar coordinates and evaluate the following integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \left(x^2 + 2xy\right) dy dx \qquad (6½)$$

(b) Find the area of the Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ using double integral. (6½)

(c) Evaluate the integral of the function $f(r, \theta, \varphi) = r^2 Cos^2\theta Sin^2\varphi$ over a sphere of radius 'a' with centre at the origin. (6½)

3. (a) Determine the nature of the stationary points of the function

$f(x,y) = x^3 + 3xy^2 - 3y^2 - 3x^2 + 4$ (5)

(b) Show that the function $e^{\alpha x}$ is an Eigen function of the operator

$\frac{d^2}{dx^2}+2\frac{d}{dx}+3$ what is the Eigen value ? (5)

(c) Determine the commutator of the operators $\hat{A}=\frac{d}{dx}-x$ and $\hat{B}=\frac{d}{dx}+x$.

(5)

4. (a) Show that the vectors a, b, c are of unit length and are mutually perpendicular, where vectors a, b, c are given by

$\vec{a}=\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}$, $\vec{b}=\frac{1}{\sqrt{6}}\hat{i}-\frac{2}{\sqrt{6}}\hat{j}+\frac{1}{\sqrt{6}}\hat{k}$ and $\vec{c}=\frac{-1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k}$. (6½)

(b) Show that $\nabla.\left(\frac{\vec{r}}{r^2}\right) = r^{-2}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = \left|\vec{r}\right|$. (6½)

(c) Show that curl $v = 0$ if v = grad f. (6½)

5. (a) Solve the following set of equations using Cramer's rule

$$2x - 3y + 4z = 8$$

$$y - 3z = -7$$

$$x + 2y + 2z = 11$$ (6½)

(b) Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$. (6½)

(c) State the condition under which a square matrix is invertible. Find the characteristic equation of the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ and hence compute its inverse if it exists. (6½)

6. (a) Solve the equation $z^6 = -1$ and plot the roots in the Complex plane. (6½)

(b) Express z, z* (conjugate of z) and z^{-1} in terms of Euler's formula where $z = 1 + i$. (6½)

(c) Define orthogonal matrix. Show that the matrix A is orthogonal

$$A = \frac{1}{9}\begin{pmatrix} 1 & 8 & -4 \\ 4 & -4 & -7 \\ 8 & 1 & 4 \end{pmatrix}$$ (6½)