This question paper contains **4** printed pages]

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *two* parts from each question.

All questions are compulsory.

Marks are indicated against each question.

Unit I

1. (*a*) Solve :   6½

$$(y \sec^2 x + \sec x \tan x)\, dx + (\tan x + 2y)\, dy = 0.$$

(*b*) Solve :   6½

$$(x^2 + y^2 + 2x)\, dx + 2y\, dy = 0.$$

(*c*) Solve :   6½

$$xp^2 - 2yp + ax = 0.$$

2.   (a)   Solve :   6½

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x} \sin x.$$

(b)   Solve :   6½

$$x^2\frac{d^2y}{dx^2} - 5x\frac{dy}{dx} + 8y = 2x^3, \ x > 0.$$

(c)   Show that the Wronskian of two solutions of the second order homogeneous linear differential equation $a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)\,y = 0$, is either identically zero or never zero on $a \le x \le b$, where a_0, a_1 and a_2 are continuous real functions on real interval $a \le x \le b$, and $a_0(x) \ne 0$ for any x on $a \le x \le b$.   6½

3.   (a)   Using method of variation of parameters, solve the differential equation :   6½

$$\frac{d^2y}{dx^2} + 4y = \sec^2 2x.$$

(b)   Given that $y = x$ is a solution of   6½

$$(x^2 - 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0,$$

find a linearly independent solution by reducing the order. Write the general solution.

(c)   A large tank initially contains 100 gal of pure water. Starting at $t = 0$, a brine containing 4 lb of dissolved salt per gallon flows into the tank at rate of 5 gal/min. The mixture is kept uniform by stirring, and the stirred mixture simultaneously flows out at the slower rate of 3 gal/min.   6½

(i)   How much salt is in the tank at the end of 20 min ?

(ii)   How much salt is present after a long time ?

4. (*a*) Solve : 6½

$$\frac{dx}{dt} + 4x + 3y = t,$$

$$\frac{dy}{dt} + 2x + 5y = e^t.$$

(*b*) Solve : 6½

$$\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}.$$

(*c*) Solve : 6½

$$zy\, dx = zx\, dy + y^2\, dz.$$

Unit II

5. (*a*) Eliminate the arbitrary function *f* from the equation :

$$z = f\left(\frac{xy}{z}\right)$$

to form the corresponding partial differential equation. 5½

(*b*) Find the general solution of the differential equation : 5½

$$(y + x)\, px = (x + y)qy - (x - y)\,(2x + 2y + z).$$

(*c*) Find the complete integral of the equation : 5½

$$xp + 3yq = 2(z - x^2 q^2).$$

6. (a) Find the complete integral of the equation : 6

$$(p^2 + q^2)y = z.$$

(b) Show that the equations : 6

$$xp = yq, \quad z(xp + yq) = 2xy$$

are compatible and find their solution.

(c) Reduce the equation : 6

$$\frac{\partial^2 z}{\partial x^2} - x^2 \frac{\partial^2 z}{\partial y^2} = 0$$

to canonical form.