[This question paper contains 4 printed pages.]

6587 *Your Roll No.* ....

B.Sc. (Hons.) Computer Science / I Sem. B

Paper – CS 102 : Discrete Structures

(Admissions of 2001 to 2009)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **all** *questions.*
Parts of a question must be answered together.

1. (a) Let L(x,y) be the statement "x loves y", where the universe of discourse for both x and y consists of all people in the world. Use quantifiers to express each of the following statements

 (i) Everybody loves somebody.

 (ii) There is somebody whom everybody loves.

 (iii) Nobody loves everybody.

 (iv) There is someone whom no one loves. (4)

 (b) Prove the conclusion: Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore there is a person in this class who cares about ocean pollution. (4)

P.T.O.

(c) Prove the tautology

$$(p \to q) \leftrightarrow (\neg q \to \neg p) \quad (3)$$

(d) Convert into PDNF

$$(q \to p) \wedge (\neg p \wedge q) \quad (3)$$

(e) From the given set of premises prove the conclusion

(i) $(p \vee q)$, $(\neg p \vee r)$, $(\neg q \vee r)$ = Conclusion : r

(ii) $\neg p$, $\neg q \vee p$, $\neg r \vee p$ = Conclusion : $\neg r$

(3+3)

2. (a) In a graph, the number of odd degree vertices is always even. Prove. (4)

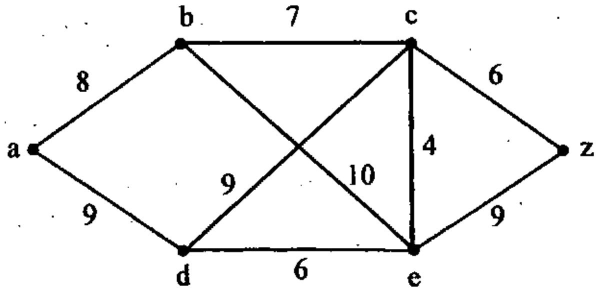
(b) How many vertices are there in a graph with 20 edges if each vertex is of degree 5 ? (2)

(c) If the graph with six edges and having degree sequence 1, 2, 3, 4, 6 can exist. (2)

(d) Define cut-set. A cut-set and any spanning tree must have at least one edge in common. Prove. (1+2)

(e) Prove that an undirected graph possesses an eulerian path if and only if it is connected and has either zero or two vertices of odd degree. (3)

3. (a) Find the shortest path from 'a' to 'z' in the graph below. Give algorithm also. (5+2)



(b) Write the algorithm for Insertion sort and prove its correctness using loop invariant. (5)

4. (a) Evaluate the product

$$\prod_{K=1}^{n} 2.4^{K}$$ (3)

(b) Show that the solution of $T(n) = 2T\left(\sqrt{\lfloor n \rfloor}\right) + \lg n$ is $O(\lg n \lg\lg n)$. (4)

(c) Let $f(n) = n^2 + 4n$ and $g(n) = n^2$, $n \geq 0$. Show that $f(n) = O(g(n))$. (3)

(d) Let A be a set with seven elements. Find the next largest permutation, in lexicographic order, after the following permutation of set A

P: 1543762 (2)

(e) Determine the number of ways in which we can make up strings of four distinct letters followed by three distinct digits. (2)

P.T.O.

5. (a) The solution of the recurrence relation

$a_r = Aa_{r-1} + B\,3^r \quad r \geq 1$ is

$a_r = C\,2^r + D\,3^{r+1} \quad r \geq 0$

Given that $a_0 = 19$ and $a_1 = 50$. Determine A, B, C, D. (6)

(b) Solve $a_r = 3a_{r-1} + 2 \quad r \geq 1,\ a_0 = 1$. Solve the given recurrence relation using generating function. (4)

6. (a) Given that

$$a_r = \begin{cases} 3 & 0 \leq r \leq 3 \\ r+2 & r \geq 4 \end{cases}$$

Find $S^{-5}a$, S^5a, Δa. (3)

(b) Rewrite the summation $\sum_{i=1}^{n} g(i-1)a^i x^{i+1}$, changing the limits of i in the summation so that i starts from-1. (2)