This question paper contains 4+2 printed pages]

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** (Section A) is compulsory.

Attempt any *four* questions from Section B.

Parts of a question should be attempted together.

Section A

1. *(a)* In a class of 25 students, 12 have taken economics, 8 have taken economics but not

   political science. Find the number of students who have taken economics and political

   science and those who have taken politics but not economics.   4

   (b) $f(x) = x^2$ and $g(x) = 2^x$

   Find $f \, o \, f(x), g \, o \, g(x), f \, o \, g(x), g \, o \, f(x)$.   4

(c) Use the master method to find the running time for the given recurrence relation : 4

$$T(n) = 125 \, T(n/5) + n^3.$$

(d) How many numbers of different messages that can be represented by sequences of three dashes and two dots ? 2

(e) Suppose that a connected planar simple graph has 20 vertices, each of degree 3. Into how many regions does a representation of this planar graph split the plane ? 3

(f) If G is a connected planar simple graph with e edges and v vertices, where $v \geq 3$, then $e \leq 3v - 6$. 5

(g) Let a be a numeric function such that : 4

$$a_r = \begin{cases} 2 & 0 \leq r \leq 3 \\ 2^{-r} + 5 & r \geq 4 \end{cases}$$

Determine Δa and ∇a.

(h) Find the particular solution for the given difference equation : 5

$$a_r + a_{r-1} = 3r \, 2^r.$$

(i) Show that :

$$\neg(p \vee (\neg p \wedge q)) \text{ and } \neg p \wedge \neg q$$

are logically equivalent by (without using truth table method). 4

Section B

2. (*a*) Give an example of a relation that is reflexive but neither symmetric nor transitive. 3

(*b*) Show that :

$$1.2.3 + 2.3.4 + 3.4.5 + ... + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}.$$ 4

(*c*) Let

$$A = \{1, 2, 3, 4\} \text{ and } B = \{a, b, c, d\}.$$

Let $f : A \rightarrow B$ such that :

$$f : 1 \rightarrow a, 2 \rightarrow a, 3 \rightarrow a, 4 \rightarrow a.$$

Verify whether f is an onto or one to one function. 3

3. (*a*) In how many ways can a cricket team of eleven be chosen out of a batch of 14 players ? How many of them will : 6

(*i*) include a particular player ?

(*ii*) exclude a particular player ?

(*b*) Show that :

$$3x^2 + 8x \log x \text{ is } \Theta(x^2).$$ 4

4. (*a*) List all the steps used to search for 9 in the sequence 1, 3, 4, 5, 6, 8, 9, 11 using :   5

   (*i*)   a linear search

   (*ii*) a binary search.

   (*b*) Give big-O estimates for the factorial function and the logarithm of the factorial function.   5

5. (*a*) Determine the discrete numeric function for the given generating function :   4

$$A(z) = 7z^2 / (1 - 2z)(1 + 3z).$$

   (*b*) Solve the recurrence relation :

$$a_r - 7a_{r-1} + 10a_{r-2} = {}_3r,$$

   given that $a_0 = 0$ and $a_1 = 1$.   6

6. (*a*) Let *p* denote the statement, "The material is interesting" and *q* denotes the statement, "The exercises are challenging" and *r* denotes the statement, "The course is enjoyable".

   Write the following statements in symbolic form :

   (*i*)   The material is interesting and the exercises are challenging.

(*ii*)   If the material is not interesting and the exercises are not challenging then the course is not enjoyable.

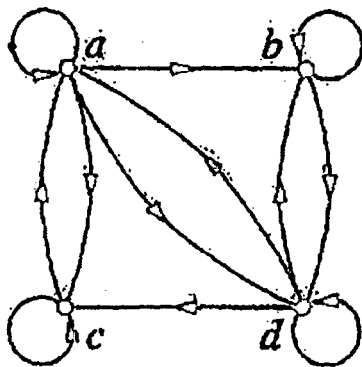(*iii*)   Either the material is interesting or the exercises are not challenging, but not both.   3

(*b*)   Given the value of $p \to q$ is true, determine the value of :
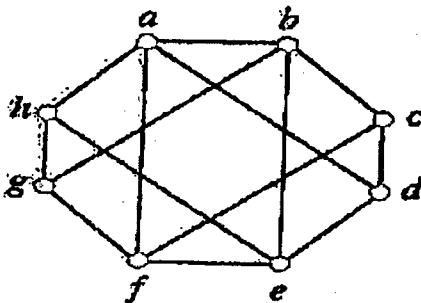
$$\bar{p} \vee (p \leftrightarrow q).$$ 3

(*c*)   Show that $\bar{p}$ is tautologically implied by :

$$\overline{(p \wedge \bar{q})}, \bar{q} \vee r, \bar{r}.$$ 4

7.   (*a*)   Find the adjacency matrix for the given directed multigraph.   4

(b) Determine whether the given graph is planar, if so, draw it so that no edges cross.   3

(c) How many vertices does a full 5-ary tree with the 100 internal vertices have ?   3