

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 772

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Your Roll No.....

Unique Paper Code : 234103

Name of the Paper : Discrete Structures (CSHT-102)

Name of the Course : **B.Sc. (H) Computer Science**

Semester : I

Duration : 3 Hours

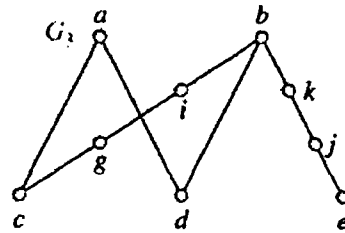
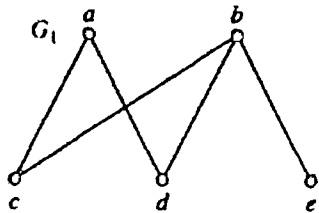
Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **Section A** is compulsory.
3. Attempt any 4 questions from **Section B**.
4. Please attempt parts of a question together.

SECTION A

1. (a) Show that the graphs G_1 and G_3 (as given) are homeomorphic. (3)



- (b) Can there be a complete graph with 20 vertices and 180 edges? Justify your answer. (3)

- (c) Draw the graph G whose adjacency matrix $A = [a_{ij}]$ is given as

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \quad (3)$$

P.T.O.

- (d) Determine the discrete numeric function for the given generating function

$$A(x) = \frac{2 + 3z - 6z^2}{1 - 2z} \quad (3)$$

- (e) Given that the value of $p \rightarrow q$ is true, can you determine the value of $\bar{p} \vee (p \leftrightarrow q)$? (3)

- (f) Let $A = \{a, b, c\}$ be a set and the relation R on A is defined as :

$$R = \{(a, a), (a, b), (b, c), (c, c)\}.$$

Is R (i) reflexive, (ii) symmetric, and (iii) transitive ? (3)

- (g) Determine the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7. (4)

- (h) Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent using the laws of logical equivalence. (4)

- (i) Find $f + g$ and fg for the functions f and g where $f(x) = x^2 + 1$ and $g(x) = x + 2$. (4)

- (j) Use master method to find the running time for the given $T(n) = 124 T(n/5) + n^3$. (5)

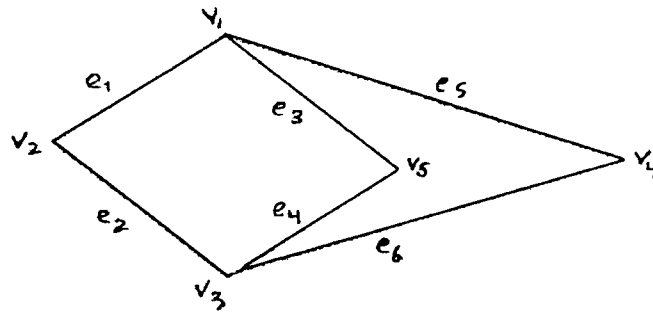
SECTION B

2. (a) A candidate is required to answer 6 out of 10 questions which are divided into two groups containing 5 questions each and he is permitted to attempt not more than four questions from any group. In how many ways can he select the questions ? (5)

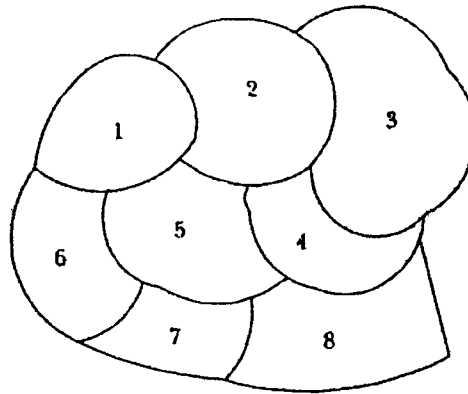
- (b) Prove by the principle of mathematical induction that

$$P(n) : 1 \cdot 2 + 2 \cdot 2^2 + \dots + n \cdot 2^n = (n - 1) 2^{n+1} + 2 \quad (5)$$

3. (a) Determine whether the given graph is bipartite or not. Justify your answer. (3)



- (b) Show that if 11 numbers are chosen from the set $\{1, 2, \dots, 20\}$, then one of them will be multiple of another. (2)
- (c) Define the chromatic number of a graph G . Find $\chi(G)$. (5)



4. (a) Let D_n denote the set of all positive divisors of n . Draw the Digraph and Hasse Diagram for the poset (D_{30}, \leq) . (5)
- (b) Given that $a_0 = 0$, $a_1 = 1$, $a_2 = 4$ and $a_3 = 12$. Satisfy the recurrence relation
- $$a_r + C_1 a_{r-1} + C_2 a_{r-2} = 0.$$
- Determine a_r . (5)
5. (a) Show that \bar{p} is tautologically implied by $\overline{p \wedge \bar{q}}$, $\bar{q} \vee r$, \bar{r} . (4)
- (b) Suppose that the number of bacteria in a colony triples every hour.
- (i) Set up a recurrence relation for the number of bacteria after n hours have elapsed.

- (ii) If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours ? (4)
- (c) Give the converse and the inverse of the statement "If it rains today, then I will drive to work". (2)
6. (a) List all the steps used to binary search for 9 in the given sequence of numbers 1, 3, 4, 5, 6, 8, 9, 11, 13, 15, 17. (5)
- (b) Define big O notation. Prove that $f(n) = n^2 + 4n$ and $g(n) = n^2$, $n \geq 0$. (5)
7. (a) Show that the following premises are inconsistent :- (5)
- (i) If jack misses many classes through illness, then he fails high school.
 - (ii) If jack fails high school, then he is uneducated.
 - (iii) If jack reads a lot of books, then he is not educated.
 - (iv) Jack misses many classes through illness and reads a lot of books.
- (b) Suppose that $f_1(x)$ is $O(g_1(x))$ and $f_2(x)$ is $O(g_2(x))$. Then $(f_1 + f_2)(x)$ is $O(\max(|g_1(x)|, |g_2(x)|))$. (5)