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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **15**

Unique Paper Code : **235166** **G**

Name of the Paper : **Maths-I Calculus and Matrices (MAPT-101)**

Name of the Course : **B.Sc. (Hons.) Computer Science/B.Sc. (Mathematical Sciences)/ B.Sc. (Physical Sciences)**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* questions from each section.

Section I

1. (*a*) Verify that the set $\{(1, -1), (1, 1)\}$ is a basis of R^2.

(*b*) Examine which of the following is a subspace of R^2 :

$$U = \{(a, b^2); a, b \in R\}$$

$$V = \{(a, b); a > 0, a, b \in R\}.$$

(*c*) Which of the following transformations are linear ? Also justify :

(1) $T : R^2 \to R^2$ defined as $T(x, y) = (1, 2)$

(2) $T : R^3 \to R^2$ defined as $T(x, y, z) = (x, 4y)$. 4,4,4

2. (*a*) Solve the following system of equations using elementary row operations :

$$2x + 3y + z = 9$$

$$x + 2y + 3z = 6$$

$$3x + y + 2z = 8.$$

P.T.O.

(*b*) Reduce the matrix :

$$\begin{pmatrix} 2 & 3 & 3 \\ 3 & 6 & 12 \\ 2 & 4 & 8 \end{pmatrix}$$

to a riangular form by elementary row operations and hence determine its rank.

(*c*) Find Eigen values and Eigen vectors corresponding to one of them for the matrix :

4,4,4

$$\begin{pmatrix} 1 & 3 & 1 \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{pmatrix}.$$

3. (*a*) For what values of λ and μ do the following system of equations :

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

have an infinite number of solutions.

(*b*) Find the inverse of the matrix using E-row operations :

$$\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}.$$

(*c*) Find the value of c for which the vectors $\{(1, 4), (c, -8)\}$ are linearly dependent.

4,4,4

Section II

4. (*a*) Use the definition to show that the sequence $\left(2+\left(-\frac{1}{2}\right)^n\right)$ converges to 2.

(*b*) Sketch the graph of $y = \sin(2x + 1)$. Mention the transformation used at each step.

(*c*) Find the *n*th derivative of :

$$y = \frac{x}{1 + 3x + 2x^2}.$$

6,6,6

5. (*a*) Assume that the rate at which radioactive nuclei decay is proportional to the number of nuclei present in a given sample. In a certain sample, 10% of the original number of radioactive nuclei has undergone disintegration in a period of 200 years. Find the percentage of the original radioactive nuclei that will remain after 1000 years.

(*b*) Show that :

$$w(x, t) = 4\cos(2x + 2ct) + e^{x+ct}$$

satisfies wave equation.

(*c*) If $u = f(r)$, where

$$r = \sqrt{x^2 + y^2},$$

prove that : 6,6,6

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r).$$

6. (*a*) If

$$y = e^{m \cos^{-1} x},$$

show that :

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0.$$

(*b*) Find the Maclaurin's series expansion for $f(x) = \sin x$, assuming that :

$$\lim_{n\to\infty} R_n(x) = 0.$$

(*c*) Sketch the level curves of the function $f(x, y) = 10 - x^2 - y^2$ of height 1, 6 and 10. 6,6,6

Section III

7. (*a*) Prove that for any *two* complex numbers z_1 and z_2 :

$$|z_1 + z_2| \leq |z_1| + |z_2|.$$

(*b*) Let z_1, z_2, z_3 be complex numbers such that :

$$z_1 + z_2 + z_3 = 0 \text{ and } |z_1| = |z_2| = |z_3| = 1.$$

Prove that :

$$z_1^2 + z_2^2 + z_3^2 = 0.$$

4,3½

8. (*a*) Use De Moivre's theorem to solve the equation :

$$z^7 - z^4 + z^3 - 1 = 0.$$

(*b*) Prove that :

$$\left(\frac{1 + \sin \phi + i \cos \phi}{1 + \sin \phi - i \cos \phi}\right)^n = \cos\left(\frac{n\pi}{2} - n\phi\right) + i \sin\left(\frac{n\pi}{2} - n\phi\right).$$

4,3½

9. (*a*) Form an equation in the lowest degree with real coefficients which has $2 - 3i$ and $3 + 2i$ as two of its roots.

(*b*) Find all the values of $\left(\sqrt{3} - i\right)^{2/5}$. 4,3½