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Your Roll No. ....

1784

B.Sc. (Hons.) Computer Science/II Sem. A

Paper CS-204 – PROBABILITY

(Admission of 2001 and onwards)

Time : 3 Hours **Maximum Marks : 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **all** questions.

All questions carry equal marks.

1. Consider two urns. The first contains two white and seven black balls, and the second contains five white and six black balls. A fair coin is flipped and then a ball is drawn from the first urn or the second urn depending on whether the outcome was heads or tails. What is the conditional probability that the outcome of the toss was heads given that a white ball was selected ? 5

2. Give an example to illustrate that pairwise independence does not imply joint independence. 5

3. If the density function of X equals

$$f(x) = \begin{cases} ce^{-2x} & , \quad 0 < x < \infty \\ 0 & , \quad x < 0 \end{cases}$$

Find C. What is $P(X > 2)$? 5

4. Let X and Y each take on either the value 1 or –1. Let 5

$P(1, 1) = P(X = 1, Y = 1)$

$P(1, -1) = P(X = 1, Y = -1)$

$P(-1, 1) = P(X = -1, Y = 1)$

$P(-1, -1) = P(X = -1, Y = -1)$

Suppose that $E(X) = E(Y) = 0$. Show that

(a) $P(1, 1) = P(-1, -1)$

(b) $P(1, -1) = P(-1, 1)$

Let $P = 2P(1, 1)$. Find

(c) Var(X)

(d) Var(Y)

(e) Cov(X, Y)

5. Find m.g.f. of Poisson distribution with parameter λ. Use it to find the mean and variance of Poisson distribution. 5

6. Suppose that $X_1, X_2, \ldots\ldots, X_n$ are independent and identically distributed random variables with mean μ and variance σ^2. Prove the following : 5

(a) $E[\overline{X}] = \mu$

(b) $Var(\overline{X}) = \frac{\sigma^2}{n}$

(c) $Cov(\overline{X}, X_i - \overline{X}) = 0; i = 1, 2, \ldots\ldots, n.$

7. State and prove Markov's inequality and hence obtain Chebyshev's inequality. 5

8. Suppose the joint density of X and Y is given by

$$f(x, y) = \begin{cases} 6xy(2-x-y) & , \quad 0 < x < 1 , \ 0 < y < 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Compute the conditional expectation of X given that Y = y, where 0 < y < 1. 5

9. Suppose X is a Poisson random variable with mean λ. The parameter λ is itself a random variable whose distribution is exponential with mean 1. Show that $P\{X = n\} = \left(\frac{1}{2}\right)^{n+1}$. 5

10. Let X_1 and X_2 be independent geometric random variables with respective parameters P_1 and P_2. Find $P\{|X_1 - X_2| \le 1\}$ 5

11. Prove that : 5

$E[X] = E[E[X \mid Y = y]]$

12. A particle moves on a circle through points which have been marked 0, 1, 2, 3, 4 (in a clockwise order). At each step, it has probability 'P' of moving to the right (clockwise) and (1 – P) to the left (counter clockwise). Let X_n denote the location on the circle after n^{th} step. The process $\{X_n; n \ge 0\}$ is a Markov chain.

(a) Find the transition probability matrix.

(b) Calculate the limiting probabilities. 5

13. Let $\{X_n; n \geq 0\}$ be a Markov chain having state space $S = \{1, 2, 3, 4\}$ and transition matrix. **5**

$$P = \begin{Bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{Bmatrix}$$

Examine the nature of states of above Markov Chain.

14. Let X and Y denote independent unit normal random variables. Show that **5**

$R^2 = X^2 + Y^2$ and $\theta = \tan^{-1}\left(\frac{Y}{X}\right)$ are independent. Identify the distributions of R^2 and θ and use it to generate random numbers from chi-square distribution.

15. Two experiments of the following nature are performed. Two shots are fired at one target, the target having a probability of $\frac{1}{2}$ of being hit on each shot. For the second experiment four shots are fired at another target, this target having a probability of $\frac{1}{4}$ of being hit on each shot. Which experiment has less uncertainty associated with it ? **5**
