This question paper contains 2 printed pages.

1783 Your Roll No..............................

B.Sc. (Hons.) II Sem. / Computer Science **A**

Paper 203 - Calculus II
(Admissions of 2001 and onwards)

Time : 3 hours *Maximum Marks : 70*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.
All questions carry equal marks.
Use of calculator is permitted.

1. a) Use Max - Min inequality to find upper and lower bounds for the value of $\int_0^1 \frac{1}{1+x^2}\,dx$. Find upper and lower bounds for $\int_0^{0.5} \frac{1}{1+x^2}\,dx$ and $\int_{0.5}^{1} \frac{1}{1+x^2}\,dx$. Add these to arrive at improved estimates of $\int_0^1 \frac{1}{1+x^2}\,dx$

2 Express the solutions of the initial value problems in terms of integrals.

a) $\frac{dy}{dx} = \sqrt{1+x^2} \quad y(1) = -2$

b) $\frac{dy}{dx} = g(t) \quad y(t_0) = y_0$

3. Find the volume using slicing method of a solid that lies between planes perpendicular to the x-axis and x = -1 and x = 1. The cross sections perpendicular to the x - axis are circular disks whose diameters run from the parabola $y = x^2$ to the parabola $y = 2 - x^2$.

4. Find the surface of the cone frustrum generated by revolving the line segment $y = \frac{x}{2} + \frac{1}{2}$, $1 \le x \le 3$, about the y - axis.

5. A particle moves on a cycloid in the xy plane in such a way that its position at time 't' is $\vec{r}(t) = (t - \sin t)\,\vec{i} + (1 - \cos t)\,\vec{j}$. Find the maximum values of $|\vec{v}|$ and $|\vec{a}|$

6. Find the centroid of the region in the first quadrant bounded in the first quadrant bounded by the x - axis, the parabola $y^2 = 2x$ and the line $x + y = 4$.

OR

Find the polar moment of inertia about the origin of a thin plate of density $\delta(x, y) = 1$ bounded by the quarter circle $x^2 + y^2 = 1$ in the first quadrant.

7. Find the volume of the region D enclosed by the surfaces $Z = x^2 + 3y^2$ and $Z = 8 - x^2 - y^2$.

8. Find the average value of $F(x, y, z) = x + y - z$ over the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 1$, $y = 1$ and $z = 2$.

9. Find the average height of hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ above the disk $x^2 + y^2 \leq a^2$ in the XY plane.

10. Find the Fourier series of the function

$$f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases}$$

$$\text{and } f(x + 2\pi) = f(x)$$

Hence obtain

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots\ldots\ldots\ldots = \pi/4$$

11. Show that an analytic function of constant absolute value is constant.

12. Write a short note on e^Z. Explain where the line x = constant and y = constant will be mapped onto. What is the fundamental region of e^Z?

13. Find a linear fractional transformation that maps $2i, -2, -2i$ onto $-2, -2i, 2$ respectively.

14. State Cauchy's Integral Formula and evaluate

$$\oint_C \frac{Z - 23}{Z^2 - 4Z - 5}\, d_Z \qquad C : |Z - 2| = 4$$

15. Evaluate

$$\int_0^{2\pi} \frac{\cos\theta}{13 - 12\cos 2\theta}\, d\theta$$

using calculus of residues.