Sr. No. of Question Paper : 1549   **C**   **Roll No**.............

Unique Paper Code   : 235266

Name of the Course   : B.Sc. (Hons.) Computer Science I / B.Sc. Mathematical Sciences I / B.Sc. Physical Sciences I

Name of the Paper   : Calculus and Geometry (MAPT 202)

Semester   : II

Duration   : 3 Hours   Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All the questions are compulsory.

3. Attempt any **two** parts from each question.

4. Marks of each part are indicated.

1. (a) Find the points of discontinuity of the function

$$f(x) = \begin{cases} \dfrac{x^4 + x^3 + 2x^2}{\tan^{-1} x} & ; \; x \neq 0 \\ 0 & ; \; x = 0 \end{cases}$$

     (6)

   (b) Define the relationship between continuity and uniform continuity. Give example to illustrate the same. (6)

   (c) Find $\dfrac{dy}{dx}$ for $e^{xy} = xy$. (6)

2. (a) State Cauchy's Mean Value Theorem and also interpret it geometrically. (6)

   (b) Using $\varepsilon - \delta$, prove that $\lim\limits_{x \to 3} x^2 = 9$. (6)

   (c) Find the asymptotes to the curve $(x^2 + y^2) x - ay^2 = 0$. (6)

3. (a) Determine the intervals of concavity & convexity and points of inflexion for the curve $y = x^4 - 4x^3 - 18x^2 + 1$. (7)

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(b) Determine the position and nature of double points of the curve

$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0$ (7)

(c) Trace the curve $x(x^2 + y^2) = a(x^2 - y^2)$. (7)

4. (a) Trace the curve $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$. (6)

(b) Obtain reduction formula for $\int \cos^n x\, dx$.

Hence. evaluate $\int \cos^4 x\, dx$. (6)

(c) Find the area between the curve $y^2(2a - x) = x^3$ and it's asymptote. (6)

5. (a) Find the volume of the solid obtained by revolving the cardioid
$r = a(1 + \cos\theta)$ about the initial line. (6)

(b) Describe the graph of $x = y^2 - 4y + 2$, label it and write reflection property for the graph and draw it on the graph. (6)

(c) Find the equation of the ellipse with foci $(1,2)$ and $(1,4)$, minor axis of length 2 units. (6)

6. (a) Rotate the co-ordinate axis to remove xy term and sketch the curve $xy = -9$. (6½)

(b) (i) Find the equation of the sphere that is circumscribed about the cube that is centered at the point $(-2,1,3)$ and has sides of length 1 unit that are parallel to the co-ordinate planes.

(ii) $\dfrac{d\left[r(t) \times r'(t)\right]}{dt} = r(t) \times r''(t)$ (3½+3)

(c) (i) Sketch the graph of

$\dfrac{x^2}{4} + \dfrac{y^2}{16} + \dfrac{z^2}{9} = 1$

Show the points of intersection with the 3 co-ordinate axes.

(ii) Find curl **F** where

$F(x,y,z) = \dfrac{x\,i + y\,j + z\,k}{\sqrt{x^2 + y^2 + z^2}}$ (3½+3)