{This question paper contains 3 printed pages}

Roll No._____

Serial Number of the question paper: **1942** **C**

Name of the course: B.Sc (Hons.) Computer Science

Name/Title of the Paper: CS-203: Calculus II

Semester: II Semester

Duration: 3 hours Max. Marks: 75

Attempt all questions. All questions carry equal marks. Use of nonprogrammable scientific calculator is allowed.

1. Show that

$$f(x) = \begin{cases} 1 & \text{when } x \text{ is rational} \\ -1 & \text{when } x \text{ is irrational} \end{cases}$$

is not Riemann integrable in [0,1].

2. Solve the initial value problem

$$\frac{dy}{dx} = \tan x$$

Initial condition

$$y(1) = 5$$

3. The velocity of a particle moving in space is

$$\frac{d\vec{r}}{dt} = (t^2 - 4t)\hat{i} + t\hat{j} - 2t^2\hat{k}$$

Find the particle's position as a function of t if

$$\vec{r} = \hat{i} + \hat{j} \quad \text{when} \quad t = 0.$$

4. The region in the first quadrant enclosed by the parabola $= x^2$, the y - axis and the line y=1 is revolved about the line $x = 3/2$ to generate a solid . Use washer's method to find the volume of the solid.

5. Find the area of the surface generated by revolving the curve $y = x^2, \quad 0 \le x \le 1/2$ about the x-axis.

6 Evaluate the following integral by changing the order of integration

$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y \, dx \, dy$$

7 Find the average value of the function $f(r,\theta,z) = r$ over the region bounded by the cylinder r = 1 between the planes z = -1 and z = 1.

8 Evaluate the integral by changing to spherical coordinates

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{1} dz \, dy \, dx$$

9. Prove that $u(x,y) = e^{-x}(x \sin y - y \cos y)$ is a harmonic function. Find v such that $f(z) = u + iv$ is analytic.

10. Find the linear fractional transformation that maps $z_1 = +i$, $z_2 = -i$, $z_3 = 0$ onto $w_1 = 0$, $w_2 = i$, $w_3 = \infty$ respectively.

11. Find the image of $x = constant$ and $y = constant$ under $f(z) = \sin z$.

12. Evaluate $\oint_c \frac{e^{2z}}{(z-1)^4}$, where c is the circle $|z| = 3$.

13. State Laurent's theorem and obtain the first four terms of the Laurent's series expansion of

$$f(z) = \frac{e^z}{z(z^2-1)}$$

valid for $0 < |z| < 1$.

14. Find the Fourier series of the periodic function f with period 2π, defined as

$$f(x) = \begin{cases} -1 & -\pi < x \le 0 \\ 1 & 0 < x \le \pi \end{cases}$$

15. Find the Fourier series of the function x^2, periodic with period $2l$ on the interval $[-l, l]$.

1979

B.Sc. (Hons.) Computer Science / III Sem.

Paper 304 : STATISTICS

C

Time:3 Hours Max. Marks : 75

Attempt any ***Ten*** questions. ***All*** questions carry equal marks.
Use of scientific calculator and statistical tables is allowed.

Q1) The mean and standard deviation of a series of 17 items are 25 and 5 respectively. While calculating these measures a measurement 53 was wrongly copied as 35. Find the corrected mean and standard deviation .

Q2) The first four central moments of a distribution are 0, 2.5, 0.7 and 8.75. Comment on the skewness and kurtosis of the distribution.

Q3) Find the missing frequencies(x, y and z) in the following distribution if N is 100 and the median is 30.

Marks	No. of students	Marks	No. of students
0-10	10	30-40	30
10-20	x	40-50	z
20-30	y	50-60	10

Q4) Following are the marks obtained by students A and B in 10 sets of examination :

Sets	1	2	3	4	5	6	7	8	9	10
A's marks	44	80	76	48	52	72	68	56	60	64
B's marks	48	75	54	60	63	69	72	51	57	56

If the consistency of performance is the criterion for awarding the prize, then who should get the prize ?

Q5) From the following data calculate the rank correlation coefficient.

X	15	20	28	12	40	60	20	80
Y	40	30	50	30	20	10	30	60

Q6) Find the means of X and Y variables and the coefficient of correlation between them from the following two regression equations:

$$2Y - X = 50$$
$$3Y - 2X - 10 = 0$$

Q7) If in a trivariate distribution $r_{12} = 0.863$, $r_{13} = 0.648$, $r_{23} = 0.709$, find $r_{12\,3}$ and $R_{1.23}$.

Q 8) Given the joint density

$$f(x,y) = \frac{2\,(2x+3y)}{5} \quad ,0 < x < 1\,;0 < y < 1;$$
$$= 0, \qquad\qquad \textit{elsewhere}.$$

Find the regression equation of Y on X.

Q9) If $X_1, X_2, \ldots\ldots, X_n$ constitutes a random sample of size n from normal population with mean μ and variance σ^2, find the joint maximum likelihood estimate of these two parameters.

Q10) Let $X_1, X_2, \ldots\ldots, X_n$ denote a random sample from $U(0,\beta)$) then show that $\frac{(n+1)\,Y_n}{n}$ is an unbiased estimator of β where Y_n is the n^{th} order statistic.

Q11) A random sample of 100 recorded deaths in U.S during past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this indicate that mean life span today is greater than 70 years ? Use a 0.05 level of significance.

Q12) Define the terms Type I error, Type II error, null hypothesis, simple hypothesis and level of significance.

Q13) A random sample of 10 boys had the following IQs : 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean IQ of 100? (Given $t_{0.05}(9) = 2.262$).

Q14) Two sample polls of votes for two candidates A and B for a public office are taken, one from among the residents of rural areas and other from urban areas. The results are given in the following table:

Area	Votes for A	Votes for B	Total
Rural	620	380	1000
Urban	550	450	1000
Total	1170	830	2000

Examine whether the nature of the area is related to voting preference in this election.

(Given $\chi^2_{0.05}(1) = 3.841$).

Q15) A random variable has a Weibull distribution with parameters α and β and density function

$$f(x) = \alpha\,\beta\,x^{\beta-1}e^{-\alpha x^{\beta}} \quad , x > 0, \alpha > 0, \beta > 0;$$
$$= 0 \qquad\qquad \textit{elsewhere}.$$

Find the mean and variance of random variable X.