Sr. No. of Question Paper : 43   E   **Your Roll No**...............

Unique Paper Code   : 235266

Name of the Course   : **B.Sc. (Hons.) Computer Science / B.Sc. Mathematical Sciences / B.Sc. Physical Sciences**

Name of the Paper   : Calculus and Geometry (MAPT–202)

Semester   : II

Duration : 3 Hours   Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** the questions are compulsory.
3. Attempt any **two** parts from each question.
4. Marks of each part are indicated.

1. (a) Let f be a function defined on R by setting

$$f(x) = \begin{cases} x^2 \sin\dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that f is derivable at x = 0, and that f′ is not continuous at x = 0.

(6)

(b) Show that the function f defined by $f(x) = x^2$ is uniformly continuous in]–2, 2[.

(6)

(c) Use (ε, δ) definition to show that

$$\lim_{x \to 4} \sqrt{x} = 2$$

(6)

2. (a) Show that the function f defined as follows :

$$f(x) = \begin{cases} \dfrac{|x-2|}{x^2-4}, & x \neq 2 \\ \dfrac{1}{4}, & x = 2 \end{cases}$$

is discontinuous at x = 2. State the kind of discontinuity.

(6)

(b) State and prove Lagrange's Mean Value Theorem and give its geometrical interpretation. (6)

(c) Find the asymptotes of the curve
$xy(x^2 - y^2) + 2x^2 + 2xy + 2y^2 - 1 = 0.$ (6)

3. (a) Find the position and nature of multiple points of the curve
$$x^3 + y^3 + 3(x^2 + y^2) = 3xy + 1.$$
Also, find the equations of the tangents at each multiple point. (7)

(b) Examine the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ for concavity and convexity. Also, determine its points of inflexion. (7)

(c) Trace the curve $y^2(a + x) = x^2 (3a - x).$ (7)

4. (a) Trace the curve $x = a\cos^3\theta, \; y = a\sin^3\theta.$ (6)

(b) If $I_{m,n} = \int x^m (\log x)^n dx$, show that

$(m + 1)I_{m,n} = x^{m+1} (\log x)^n - nI_{m,n-1}$. Find the value of $\int_0^1 x^4 (\log x)^3 dx$. (6)

(c) Find the area of a loop of the curve $r = a \sin 2\theta.$ (6)

5. (a) Find the volume of the solid generated by rotating the ellipse $4x^2 + y^2 = 4$ about the x-axis. (6)

(b) Describe the graph of the equation $4y^2 - x^2 + 40y - 4x = -60.$ (6)

(c) Find the equation of the ellipse whose foci are (2,1) and (2, -3) and major axis is of length 6. Also, write its reflection property. (6)

6. (a) Rotate the axes of coordinates to get rid of the xy-term from the equation
$9x^2 - 24xy + 16y^2 - 80x - 60y + 100 = 0$ and trace the conic. (6.5)

(b) For $F = F(x,y,z)$, $G = G(x,y,z)$, $\varphi = \varphi(x,y,z)$, prove that
$\text{curl}(\varphi F) = \varphi\, \text{curl}F + \nabla\varphi \times F.$ (6.5)

(c) (i) Find $\text{curl}\left(\text{curl}\,\vec{A}\right)$ if $\vec{A} = y^2x\,\hat{i} - 3yz\,\hat{j} + xy\,\hat{k}.$ (3)

(ii) Find the center and radius of the sphere
$x^2 + y^2 + z^2 - 2x - 4y + 8z + 17 = 0.$ (3.5)