

This question paper contains 4 printed pages]

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S. No. of Question Paper : 48

Unique Paper Code : 235466

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Name of the Paper : MAPT-404 : Differential Equations

Name of the Course : B.Sc. (H) Comp. Sc., B.Sc. (Appl. Phy. Sc.) Analytical
Chemistry/Industrial Chemistry/B.Sc. Mathematical
Science/B.Sc. Physical Science

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt two parts from each question.

All questions are compulsory.

Marks are indicated against each question.

Unit I

1. (a) Solve :

6½

$$(y \sec^2 x + \sec x \tan x) dx + (\tan x + 2y) dy = 0.$$

(b) Solve :

6½

$$(x^2 + y^2 + 2x) dx + 2y dy = 0.$$

(c) Solve :

6½

$$xp^2 - 2yp + ax = 0.$$

P.T.O.

2. (a) Solve :

6½

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x} \sin x.$$

- (b) Solve :

6½

$$x^2 \frac{d^2y}{dx^2} - 5x \frac{dy}{dx} + 8y = 2x^3, x > 0.$$

- (c) Show that the Wronskian of two solutions of the second order homogeneous linear differential equation $a_0(x) \frac{d^2y}{dx^2} + a_1(x) \frac{dy}{dx} + a_2(x) y = 0$, is either identically zero or never zero on $a \leq x \leq b$, where a_0 , a_1 and a_2 are continuous real functions on real interval $a \leq x \leq b$, and $a_0(x) \neq 0$ for any x on $a \leq x \leq b$.

6½

3. (a) Using method of variation of parameters, solve the differential equation :

6½

$$\frac{d^2y}{dx^2} + 4y = \sec^2 2x.$$

- (b) Given that
- $y = x$
- is a solution of

6½

$$(x^2 - 1) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0,$$

find a linearly independent solution by reducing the order. Write the general solution.

- (c) A large tank initially contains 100 gal of pure water. Starting at $t = 0$, a brine containing 4 lb of dissolved salt per gallon flows into the tank at rate of 5 gal/min. The mixture is kept uniform by stirring, and the stirred mixture simultaneously flows out at the slower rate of 3 gal/min.

6½

(i) How much salt is in the tank at the end of 20 min ?

(ii) How much salt is present after a long time ?

4. (a) Solve :

6½

$$\frac{dx}{dt} + 4x + 3y = t,$$

$$\frac{dy}{dt} + 2x + 5y = e^t.$$

(b) Solve :

6½

$$\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$$

(c) Solve :

6½

$$zy \, dx = zx \, dy + y^2 \, dz.$$

Unit II5. (a) Eliminate the arbitrary function f from the equation :

$$z = f\left(\frac{xy}{z}\right)$$

to form the corresponding partial differential equation.

5½

(b) Find the general solution of the differential equation :

5½

$$(y + x) \, px = (x + y) \, qy - (x - y) \, (2x + 2y + z).$$

(c) Find the complete integral of the equation :

5½

$$xp + 3yq = 2(z - x^2q^2).$$

6. (a) Find the complete integral of the equation :

6

$$(p^2 + q^2)y = z.$$

- (b) Show that the equations :

6

$$xp = yq, \quad z(xp + yq) = 2xy$$

are compatible and find their solution.

- (c) Reduce the equation :

6

$$\frac{\partial^2 z}{\partial x^2} - x^2 \frac{\partial^2 z}{\partial y^2} = 0$$

to canonical form.