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6599

Your Roll No.

B.Sc. (Hons.) Computer Science / V Sem. B

**Paper 504 : Numerical Analysis and
Scientific Computing**

(Admissions of 2001 and onwards)

Time : 3 Hours

Maximum Marks : 75

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

*Attempt All questions. Parts of a question must be
answered together. Marks are indicated against
each question. Use of Non-programmable
scientific calculator is allowed.*

1. (a) Derive the following inequalities.

(i) $|e^x - e^z| \leq |x - z|$, for all $x, z \leq 0$

(ii) $py^{p-1}(x-y) \leq x^p - y^p \leq px^{p-1}(x-y)$

for $0 \leq y \leq x$, $p \geq 1$. (5)

(b) Define order of convergence of an iteration. Find the order of convergence for

$$x_{n+1} = \frac{x_n(x_n^2 + 3a)}{3x_n^2 + a}, n \geq 0$$

to compute \sqrt{a} . (5)

P.T.O.

2. (a) Give the geometric interpretation of secant method. Use secant method to find the smallest positive root of the equation :

$$x^4 - x - 10 = 0,$$

correct to three decimal places. (5)

- (b) Consider x_0, x_1 as the nodal points and ϵ the maximum value of the rounding error in the function evaluations. Show that the effect of these rounding errors on the linear interpolation is bounded by ϵ for $x_0 \leq x \leq x_1$. (5)

3. (a) For $K \geq 0$, prove that

$$f[x_0, x_1, \dots, x_K] = \frac{1}{K! h^K} \nabla^K f. \quad (5)$$

- (b) Find the minimum number of equal length subinterval needed to approximate

$$\int_1^2 x e^x dx$$

to an accuracy of atleast $\frac{1}{3} \times 10^{-6}$ using Trapezoidal rule. (5)

- (c) Evaluate

$$I = \int_0^{\pi/2} \sin x dx$$

using Gaussian-quadrature for $n = 2$. (5)

4. (a) Obtain a linear polynomial approximation to the function $f(x) = \ln x$ on the interval $[1,2]$ using the least square approximately with weight $W(x) = 1$. (5)
- (b) Solve the following system of linear equation using Crout reduction :

$$x + y + z = 1$$

$$4x + 3y - z = 6$$

$$3x + 5y + 3z = 4 \quad (5)$$

- (c) Define a diagonally dominant matrix. Solve the following system of equation using Jacobi method :

$$6x - 2y + z = 11$$

$$x + 2y - 5z = -1$$

$$-2x + 7y + 2z = 5$$

(Show three iterations). (5)

5. (a) For the differential equation

$$y' = 1 + y^2, \text{ with } y(0) = 0,$$

the starting values are

$$y(0.2) = 0.2027, y(0.4) = 0.4228, y(0.6) = 0.6841.$$

Use Adams - Moulton method to advance the solution to $x = 0.8$. (5)

- (b) Solve the following equation with boundary points $(0, 0)$ and $(2, 3.5)$ using the Rayleigh-Ritz method :

$$y'' + y = 3x^2 \quad (5)$$

6. (a) Compute the Frobenius norm and column norm for

the matrix $A = \begin{pmatrix} 5 & -5 & -7 \\ -4 & 2 & -4 \\ -7 & -4 & 5 \end{pmatrix}$. (2)

- (b) Write the codes in MATLAB/MATHEMATICA/MAPPLE for evaluating

$$\int_0^2 (3x^2 - 2x) dx$$

using Trapezoidal rule. (3)

- (c) Find the dominant eigen value and the corresponding eigen vector by the power method

$$A = \begin{bmatrix} 2 & 3 \\ 6 & 5 \end{bmatrix} \quad (5)$$

- (d) Derive the expression for $f''(x)$ using the method of undetermined coefficients. (5)