

This question paper contains 3 printed pages.]

Your Roll No.

1398

A

B.Sc. (Hons.)I

ELECTRONICS SCIENCE—Paper 1.7 (VII)

(Mathematical Physics—II)

Time : 3 Hours

Maximum Marks : 38

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

Attempt five questions in all.

Question one is compulsory.

1. (a) Test the series $\sum_{n=1}^{\infty} \frac{2^n |n|}{n^n}$ for convergence or divergence. 2
- (b) Is the set of functions e^x, e^{-x}, e^{-2x} linearly dependent? 2
- (c) State Dirichlet's theorem. 2
- (d) Write down Parseval's identity for the Fourier series. 2
- (e) Prove that the Fourier transform of the convolution of $f(x)$ and $g(x)$ is the product of their Fourier transform. 2
2. (a) Obtain the Fourier expansion of a function $f(x)$ in the interval $(-l, +l)$ 2

[P.T.O.]

- (b) Obtain the Fourier series expansion for $f(x) = x \sin x$ in the interval $-\pi < x < \pi$ and hence deduce that :

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \dots$$

5

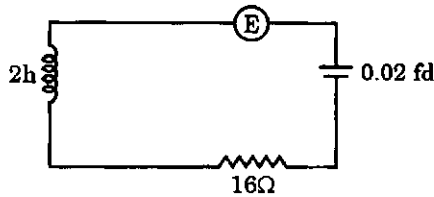
3. (a) Find the Laplace transform of Half wave rectifier function :

$$f(t) = \begin{cases} \sin \omega t & 0 < t < \pi / \omega \\ 0 & \pi / \omega < t < 2\pi / \omega \end{cases}$$

3

- (b) An inductor of 2 henrys, a resistor of 16 ohms and a capacitor of 0.02 farads are connected in series with an e.m.f. of E volts. At $t = 0$ the charge on the capacitor and current in the circuit are zero. Find the charge and current at any time $t > 0$ if (a) $E = 300$ volts, (b) $E = 100 \sin 3t$ volts by using Laplace Transform.

4



4. (a) Find the inverse Laplace transform of $\frac{S}{(S^2 + a^2)^2}$

2

- (b) Find the Fourier transform of the slit function of $f(x)$ defined as

5

$$f(x) = \begin{cases} \frac{1}{\epsilon} & : |x| \leq \epsilon \\ 0 & : |x| > \epsilon \end{cases}$$

determine the limit of the transform as $\epsilon \rightarrow 0$. Plot the function and its transform.

5. (a) Using Fourier Integral show that : 3

$$e^{-ax} = \frac{2a}{\pi} \int_0^{\infty} \frac{\cos \lambda x}{\lambda^2 + a^2} dx, a > 0, x \geq 0$$

- (b) Show that the series

$$x^2(\log 2)^q + x^3(\log 3)^q + x^4(\log 4)^q + \dots$$

is convergent when $x < 1$ and divergent when $x \geq 1$, whatever the value of q . 4

6. Find the general solution of the following differential equations :

(a) $Y'' - 2Y' + 3Y = \cos x + x^2$ 3

(b) $Y'' - 5Y' + 6Y = x \sin 3x + e^x \sin x$ 4

7. (a) Discuss the extrema of the function : 3

$$x^3 + y^3 - 63(x + y) + 12xy$$

- (b) Show that the maximum and minimum values of $x^2 + y^2$ where $ax^2 + 2hxy + by^2 = 1$ are given by the roots of the quadratic equation : 4

$$\left[a - \frac{1}{r^2} \right] \left[b - \frac{1}{r^2} \right] = h^2$$