[This question paper contains 4 printed pages.]

1006 *Your Roll No.* ....

B.Sc. (Hons.) / I **C**

ELECTRONICS – Paper 1.6 (VI)

(Mathematical Physics – I)

Time : 3 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **Five** *questions.*
Question No. **1** *is compulsory.*
Attempt at least **one** *question from each Section.*

1. Compulsory question. (5×2=10)

(a) Find an equation for the plane determined by the points $P_1(2, -1, 1)$, $P_2(3, 2, -1)$, $P_3(-1, 3, 2)$.

(b) Find grad of $1/|r|$ where, $|r| = \sqrt{(x^2 + y^2 + z^2)}$.

(c) Prove that the eigenvalues of a Hermitian matrix are real.

(d) Show divergence of a curl of a vector is zero.

(e) Define Alternating tensor. Show that $\delta_{ij}\,\varepsilon_{ijk} = 0$.

SECTION A

2. (a) State Gauss Divergence theorem. Evaluate

$$\oiint \mathbf{F}.\hat{n}\, dS$$

where $\mathbf{F} = 4xz\,\mathbf{i} - y^2\,\mathbf{j} + yz\,\mathbf{k}$ and S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$. (4)

(b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{a}, \vec{b}$ are constant vectors then show that

$$\vec{\nabla} \times \left[\left(\vec{r} \times \vec{a}\right) \times \vec{b}\right] = \vec{b} \times \vec{a}$$ (3)

3. (a) Find the unit vectors $\hat{e}_\rho, \hat{e}_\phi$, and $\hat{e}_z$ for an orthogonal cylindrical co-ordinate system. (4)

(b) Prove that the spherical coordinate system is orthogonal. (3)

SECTION B

4. (a) Define the following with an example each :

(i) Isotropic tensor

(ii) Anti-symmetric tensor (3)

(b) Establish the following using tensor notation :

(i) $A \times (B \times C) = (A.C)B - (A.B)C$

(ii) $\text{curl}\left(\phi \vec{F}\right) = \text{grad}\,\phi \times \vec{F} + \phi\,\text{curl}\,\vec{F}$ (4)

5. (a) Derive Moment of Inertia tensor and show that it is an isotropic tensor of rank two. (7)

SECTION C

6. (a) Prove that the trace of a matrix remains invariant under similarity transformation. (2)

(b) For which value of k will the vector u = (1, –2, k) in $\mathbf{R}^3$ be a linear combination of vectors v = (3, 0, –2) and w = (2, –1, –5). (2)

(c) Find the matrix representation for T

(i) standard basis

(ii) basis {[1, 1], [1, –1]} when $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + y \\ x - y \end{bmatrix}$

(3)

7. (a) Prove that the eigen values A^{-1} are the reciprocal of eigen values of a non singular matrix A. (3)

(b) Reduce the following coupled differential equation into eigen value problem and solve :

$$\frac{dy_1}{dt} = y_1 + 2y_2 - 3y_3$$

$$\frac{dy_2}{dt} = 2y_1 + 4y_2 - 6y_3$$

$$\frac{dy_3}{dt} = -y_1 - 2y_2 + 3y_3$$

Given that : $y_1(0) = 0$, $y_2(0) = 1$, $y_3(0) = 1$. (4)

(200)