[This question paper contains 4 printed pages.]

1007

B.Sc. (Hons.) / I C

ELECTRONICS SCIENCE – Paper 1.7 (VII)

(Mathematical Physics – II)

Time : 3 Hours *Maximum Marks : 38*

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

Attempt **five** *questions in all.
Question* **one** *is compulsory.*

1. Do any **five** of the following : (2×5)

   (a) Expand $f(x) = x_1^2$, for $-\pi \le x \le \pi$ as Fourier series.

   (b) State the Dirichlet conditions.

   (c) Find out whether e^x and e^{2x} are linearly dependent on each other.

   (d) Find $L^{-1} \left[\dfrac{1}{s^3 + 4s} \right]$.

   (e) State and prove the convolution theorem for Fourier transform.

P.T.O.

(f) Test for the convergence of series $\sum_{n=1}^{\infty} \dfrac{1}{\sqrt{n}}$.

2. (a) Express $f(x) = \pi - x, \ 0 < x < \pi$ as Fourier sine series.

   (b) Express $f(x) = x^2$ in the interval $-\pi < x < \pi$ as Fourier series and then evaluate

   $$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \ \cdots \tag{3+4}$$

3. (a) State and prove Fourier Integral theorem.

   (b) Write down the Fourier's cosine integral. Using this evaluate the following integral

   $$\int_0^\infty \frac{\cos \omega x}{\omega^2 + k^2} d\omega = \frac{\pi e^{-kx}}{2k} \tag{3+4}$$

4. (a) Find the transforms of

   (i) $L^{-1} \left[\log \dfrac{\left(s^2 - 1\right)}{s^2} \right]$

   (ii) $L \left[\sin 2t \sin 3t \right]$

(b) If f(t) is the periodic function with period T, then

$$\text{prove } L\left[f(t)\right] = \frac{1}{1 - e^{-sT}} \int_0^T e^{-sT} f(t) \, dt. \qquad (4+3)$$

5.   Solve following differential equations :

(a) $y'' - 5y' + 6y = e^x \cos 2x$

(b) Use the transformation $x = e^z$ and reduce the following equation :

$$x^3 y''' + 3x^2 y'' + y' = x^2 \log x$$

to one with constant co-efficients and obtain its complete solution.   (3+4)

6.   (a) Find Fourier transform of

$$f(x) = \begin{cases} 1-x^2 & \text{if } |x| \le 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$

(b) Find the maxima and minima for

$$f(x, y) = x^3 + y^3 - 63(x + y) + 12xy \qquad (4+3)$$

7.   (a) Check the convergence status of following series

$$\frac{1}{1 \times 2 \times 3} + \frac{3}{2 \times 3 \times 4} + \frac{5}{3 \times 4 \times 5} + \dots$$

(b) State the D'Alembert's Ratio test and use it to check the convergence of following series

$$\frac{1}{5} + \frac{2!}{5^2} + \frac{3!}{5^3} + \frac{4!}{5^4} + \dots \tag{3+4}$$