This question paper contains 3 printed pages. Your Roll No. ....

Sl. No. of Ques. Paper : **1779** **GC-3**

Unique Paper Code : **32511104**

Name of Paper : **Core Paper – II : Mathematics Foundation for Electronics**

Name of Course : **B.Sc. (Hons.) Electronics under CBCS**

Semester : **I**

Duration : : **3 hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt five questions in all. Q. No. 1 is compulsory.
All questions carry equal marks.
Use of non-programmable scientific calculator is allowed.

1. (a) Solve the differential equation :

$$(y + 2\sqrt{x^2 + y^2})dx - x\,dy = 0, y(1) = 0$$ 3

(b) Show that $AB = AC$ does not necessarily imply that $B = C$

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ 4 & -3 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 4 & 1 & 0 \\ 2 & 1 & 1 & 1 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 1 & -1 & -2 \\ 3 & -2 & -1 & -1 \\ 2 & -5 & -1 & 0 \end{bmatrix}$$ 3

(c) Show that gamma function is not defined for zero and negative integers. 3

(d) Show that function

$$u = \cos x . \cosh y$$

is harmonic and find its harmonic conjugate. 3

(e) Test for convergence of following series: $\sum_{n=1}^{\infty} \frac{1}{n!}$ 3

2. (a) Solve

$$(D^2 + 2)y = x^2 e^{3x} + e^x \cos 2x$$ 6

(b) Find a power series solution in power of x of the following differential equation

$$(1 - x^2)y'' - 2\,x\,y' + 2\,y = 0$$ 6

(c) Find only the indicial equation for the following differential equation using Frobenius method around $x = 0$. 3

$$x\,(1+x)y'' + (x+5)y' - 4y = 0$$

3. (a) Find eigenvalues and eigenvectors of

$$\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$ 4

(b) Verify Cayley – Hamilton theorem for matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

Find A^{-1}. Determine A^8. 5

(c) Solve the system of given equations using LU decomposition method.

$$3x_1 - 6x_2 - 3x_3 = -3$$
$$2x_1 + 0x_2 + 6x_3 = -22$$
$$-4x_1 + 7x_2 + 4x_3 = 3$$ 6

4. (a) Show that

$$A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$$

is skew-Hermitian and also unitary. 4

(b) Prove that the generating function for Bessel's functions of integral order is $e^{\frac{1}{2}x\left(t-\frac{1}{t}\right)}$ 8

(c) If

$$A = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$$

show that $(I - A)(I + A)^{-1}$ is a unitary matrix. 3

5. (a) Find the residues of following function

$$f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$$ 5

(b) Find Laurent series about the singularity indicated with the function

$$f(z) = \frac{e^{2z}}{(z-1)^3}\ ; \quad z = 1$$ 3

(c) Evaluate

$$f(z) = \frac{1}{2\pi i} \oint_c \frac{e^{zt}}{z^2\,(z^2+2z+2)}\,dz$$

around the circle c with equation $|z| = 3$. 7

6. (a) Use integral test for checking the convergence of following series

i) $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1} + \cdots$

ii) $\sin\pi + \frac{1}{4}\sin\frac{\pi}{2} + \frac{1}{9}\sin\frac{\pi}{3} + \cdots$ 3 + 4

(b) Check the convergence using ratio test of the series whose n^{th} term is

$\frac{(n+3)!}{3!n!3^n}$ 5

(b) Use Cauchy's n^{th} root test to test for convergence of following series

$1 + \frac{x}{2} + \frac{x^2}{3^2} + \frac{x^3}{4^3} + \cdots \;; x > 0$ 3

7. (a) Evaluate $\oint_c \frac{dz}{z-a}$

where c is simple closed curve and the point $z = a$ lies

i) Outside c

ii) Inside c 2+5

(b) Determine for what value of x, the following series is convergent

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{(2n-1)!}$ 4

(c)Test for convergence of following alternating series

$1 - \frac{x}{1^2} + \frac{x^2}{2^2} - \frac{x^3}{3^2} + \frac{x^4}{4^2} - \cdots$ 4