This question paper contains 7 printed pages.]

Your Roll No. ....

1386

B.Sc. (H) ELECTRONICS / II Sem. A

Paper – ELHT-201

Signals and Systems

(Admission of 2010 and onwards)

Time : 3 Hours **Maximum Marks : 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **five** questions in all including Q. No. 1 which is compulsory. Use of Scientific Calculators is allowed.

1. Attempt any **five** of the following : **5 × 3 = 15**

(a) Sketch and label the following signals :

(i) $u[n-3] - u[n-4]$

(ii) $x(t) = 4e^{-0.5\,t}\,u(t)$

(iii) $x[n] = 4\cos\left[\frac{\pi}{4}n\right]$

(b) Determine whether or not each of the following signal is periodic. If a signal is periodic, specify its time period.

(i) $x[n] = e^{j7\pi n}$

(ii) $x(t) = \sin^2 t$

(c) Show that

(i) $x[n] * \delta[n] = x[n]$

(ii) $x(t) * \delta(t - t_0) = x(t - t_0)$

(d) Determine the Fourier series representation of a continuous time periodic signal

$$x(t) = \cos 4t + \sin 6t$$

(e) Find the output y(t) using the Fourier Transform for a continuous time LTI system described by $\frac{dy(t)}{dt} + ay(t) = x(t)$ where $x(t) = e^{-at}u(t)$.

(f) In the series RLC circuit, R = 5 Ω, L = 1 H, and C = 0.5 F. There is no initial charge on the capacitor. A d.c. voltage of 50 volts is applied at t = 0. Determine the resulting current using Laplace Transform.

2. (a) Describe the basic system properties e.g. memory or memoryless system, linearity, time, invariance, causality and stability of the system. Explain in brief. 5

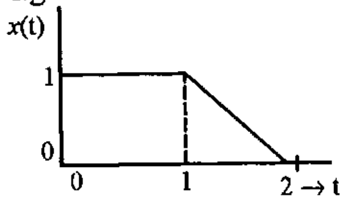
(b) Consider the system : 5

(i) $y(t) = tx(t)$

(ii) $y[n] = x[4n + 1]$

Determine whether it is memoryless, causal, linear, time invariant and stable system. Justify your answer.

(c) A continuous time signal $x(t)$ is shown in the figure below : 5



Sketch and label :

(i) $x(t + 1)$

(ii) $x\left(\frac{3}{2}t\right)$

(iii) $x(-t)$

(iv) $x(-t + 2)$

3. (a) Describe the memory and invertibility properties of a continuous time and discrete time LTI system in terms of its impulse response. 3

(b) Consider an input $x[n]$ and a unit impulse response of the discrete time system, h[n] given as **6**

$$x[n] = \begin{cases} 0.5 \;; & n = 0 \\ 2 \;; & n = 1 \\ 0 \;; & \text{otherwise} \end{cases}$$

$$h[n] = \begin{cases} 1, & n = 0, 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

Compute the output y[n] using convolution sum.

(c) Let $x(t)$ be an input to the LTI system with unit impulse response h(t) given as **6**

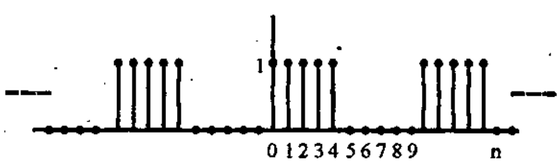
$$x(t) = e^{2t}\, u(-t)$$

$$h(t) = u(t-3)$$

Compute the output y(t) using convolution integral.

4. (a) Derive the expressions for Fourier series coefficients (exponential and trigonometric) for a discrete time periodic signal. What are the conditions for convergence of Fourier series ? **6**

(b) Determine the Fourier series coefficients for the periodic sequence $x[n]$. **6**



(c) If the Fourier series coefficient of $x(t)$ are a_k. Find the Fourier coefficients of $x(at)$ and

$$\int_{-\infty}^{t} x(t)dt.$$ 3

5. (a) Find out the transfer function H(jw) for a first order RC high pass filter and plot its magnitude and phase spectrum. Also obtain the impulse response h(t) in time domain. 6

(b) Find the Fourier transform and plot the spectrum of a signal given as $x(t) = e^{-a|t|}$ 6

(c) Compute the convolution integral for $x(t) = te^{-2t}\, u(t)$ and impulse response of the system $h(t) = te^{-4t}\, u(t)$ using Fourier transform. 3

6. (a) Derive the time shifting and time scaling properties for Laplace Transform. 4

(b) Determine the Laplace Transform and the associated region of convergence (ROC), and plot pole-zero diagram for the same. 7

$x(t) = e^{2t}\, u(-t) + e^{3t}\, u(-t)$

(c) Determine the inverse Laplace Transform of

$$X(s) = \frac{s+1}{s^2 + 2s + 10}, \text{ Real } [s] < -1. \quad 4$$

7. (a) Consider a continuous time LTI system for which the input $x(t)$ and output $y(t)$ are related by the following differential equation

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t) \quad 9$$

Let $X(s)$ and $Y(s)$ denote Laplace Transform of $x(t)$ and $y(t)$, respectively and let $H(s)$ denotes the Laplace transform of $h(t)$, the system impulse response.

(A) Determine $H(s)$ as a ratio of two polynomials in s. Sketch the pole-zero pattern.

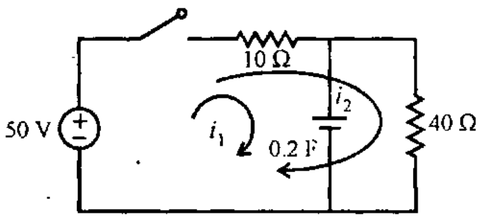
(B) Determine $h(t)$ for each of the following cases :

(i) the system is stable

(ii) the system is causal

(iii) the system is neither stable nor causal

(b) In the mesh network shown in the figure **6**



there is no initial charge on the capacitor. Find the loop currents i_1 and i_2 which result when switch is closed at $t = 0$, use Laplace Transform.