This question paper contains 4 printed pages.]

1389 *Your Roll No. ....*

B.Sc. (Hons.) Electronics / II Sem. **A**

Paper – MAHT – 204

Mathematics–I

(Admissions of 2010 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt any **two** parts from each question.*

All questions carry equal marks.

1. (a) Discuss the convergence of the sequence $\left\{\dfrac{\sin h\, n}{\cos h\, n}\right\}$. 4½

(b) Test the convergence of the series $\sum ne^{-n^2}$. 4½

(c) Test for the absolute or conditional convergence of the series $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{\sqrt{n}}$. 4½

2. (a) Without solving, show that the equation $x^4+2x^3-2=0$ has at the most one real root between 0 and 1. 4½

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(b) Separate the intervals in which the polynomial $f(x)=(4-x^2)^2$ is increasing or decreasing. 4½

(c) For every $x \geq 0$, show that

$$1+x+\frac{x^2}{2} \leq l^x \leq 1+x+\frac{x^2}{2}l^x.$$ 4½

3. (a) If $f(x,y)=\dfrac{x^2-y^2}{x^2+y^2},(x,y)\neq(0,0)$

$$=0 \quad , \quad (x,y)=(0,0),$$

then show that f is discontinuous at the origin. 4½

(b) The altitude of a right circular cone is 15 cm and is increasing at 0.2 cm/sec. The radius of the base is 10 cm and is decreasing at 0.3 cm/sec. How fast is the volume changing ? 4½

(c) If $u = \sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$, then show that ${}_{x}u_x + yu_y =$ $\tan u$. 4½

4. (a) Expand $f(x,y)=21+x-20y+4x^2+xy+6y^2$ in Taylor series of maximum order about the point (–1, 2). 4½

(b) Find the extreme value of $x^2+y^2+z^2$ subject to the condition $xyz=a^3$. 4½

(c) Trace the curve $y^2(a+x)=x^2(a-x)$. 4½

5. (a) Find the perimeter of the curve
$$r=a(\cos\theta+\sin\theta); 0\le\theta\le\pi.$$ 4½

(b) Find the volume of the solid of revolution generated by revolving the plané area bounded by the given curves $y=x^2, y=2x$ about the x-axis.

4½

(c) Find the area of the surface of revolution of the solid generated by revolving the curve whose parametric equations are $x=a\cos^3 t$, $y=a\sin^3 t$ about the x-axis. 4½

6. (a) Evaluate the double integral $\iint_R \ell^{x^2}\,dxdy$, where the region R is given by $R: 2y\le x\le 2$ and $0\le y\le 1$. 5

(b) Evaluate $\int_0^1\int_0^x\int_0^{x+y}(x+y+z)\,dz\,dy\,dx$. 5

(c) Change the variables, if necessary to evaluate $\iint_R \ell^{2(x^2+y^2)}\,dxdy$, where R is bounded by R

$R: x^2+y^2=4, x^2+y^2=25, y\le x, x\ge 0, y\ge 0.$ 5

7. (a) Find the directional derivative of the scalar function $\phi(x, y, z) = x^2 y - y^2 z - xyz$ at $(1, -1, 0)$ in the direction $\hat{i} - \hat{j} + 2\hat{k}$. 5

(b) Find the Jacobian of x, y, z with respect to cylindrical coordinates. 5

(c) Let $f(x, y, z)$ be a solution of the Laplace equation $\nabla^2 f = 0$. Then, show that ∇f is a vector which is both irrotational and solenoidal. 5

8. (a) Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the space curve $C: x = 2t^2, y = t, z = 4t^2 - t, 0 \le t \le 1$. 5

(b) Using Green's theorem, or otherwise, evaluate the integral $\oint_C \vec{F} \cdot d\vec{r}$ counter-clockwise around the boundary C of the region R, where $\vec{F} = \left[x^2 e^y, y^2 e^x\right]$, C is the rectangle with vertices (0, 0), (2, 0), (2, 3), (0, 3). 5

(c) Using Divergence Theorem, or otherwise, evaluate $\iint_S (7x\hat{i} - z\hat{k}) \cdot \hat{n}\, dA$ over the sphere $S: x^2 + y^2 + z^2 = 4$. 5