This question paper contains 4 printed pages]

Roll No. | | | | | | | | | |

S. No. of Question Paper : **1800**

Unique Paper Code : **235271**

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Name of the Paper : **MAHT-204 : Mathematics-I**

Name of the Course : **B.Sc. (Hons.) (Electronics)**

Semester : **II**

Duration : **3 Hours** Maximum Mark

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are **7** questions in this question paper. Question No. **1** is compulsory. Attempt

four questions from Question No. **2** to Question No. **7**.

1. (*a*) Define convergence of a sequence (a_n). Discuss the convergence of the sequence where $a_n = \sqrt{n+1} - \sqrt{n}$.

(*b*) If $u = x^3 y e^z$, where $x = t$, $y = t^2$, $z = \text{Log}_e t$, find $\frac{du}{dt}$ at $t = 2$.

(*c*) Determine whether the following functions are functionally dependent. Find the func relation between them in case they are functionally dependent.

$$u = \frac{x}{y}, \; v = \frac{x+y}{x-y}$$

(*d*) Evaluate $I = \int_0^{2\pi} \int_{a \sin\theta}^{a} r \, dr d\theta$.

(*e*) Define divergence of a vector function $\bar{A}(x, y, z)$. Give its physical meaning, and evaluate divergence of $(2x^2z\hat{i} - xy^2z\hat{j} + 3yz^2\hat{k})$ at the point (1, 1, 1). (3×5)

2. (*a*) Show that the sequence (a_n) defined by setting $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{n}$ does not converge.

(*b*) Test for convergence the following series $\sum u_n$ whose *n*th term is given by :

(*i*) $u_n = \frac{1}{3^n + x}\ \forall\ x > 0$; 2

(*ii*) $u_n = \frac{1}{n^2 \log n}$; 2

(*iii*) $u_n = \cos\left(\frac{1}{n}\right)$. 1

(*c*) State Rolle's theorem. Without solving show that the equation $x^4 + 2x^3 - 2 = 0$ has one and only one real root between 0 and 1. (5×3)

3. (*a*) If $u = x^3y^2\sin^{-1}(y/x)$ then show that :

$$xu_x + yu_y = 5u \quad \text{and} \quad x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 20u.$$

(*b*) Find the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$.

(*c*) Trace the curve $y^2x^2 = x^2 - a^2$. 5×3

4. (a) Find the length of the arc of the parabola $y^2 = 12x$ cut-off by its latus rectum.

(b) Find the volume bounded by the elliptic paraboloids $z = x^2 + 9y^2$ and $z = 18 - x^2 - 9y^2$.

(c) The arc of the cardioid $r = a(1 + \cos\theta)$ included between $\theta = -\pi/2$ to $\theta = \pi/2$ is rotated about the line $\theta = (\pi/2)$. Find the volume of the solid of revolution.

5×3

5. (a) Find the directional derivative of $f(x, y, z) = 4e^{2x - y + z}$ at the point (1, 1, –1) in the direction towards the point (–3, 5, 6).

(b) Find $f(r)$ such that $\nabla f = \frac{\vec{r}}{r^5}$ and $f(1) = 0$

(c) If $\overline{F} = (y - 2x)\hat{i} + (3x + 2y)\hat{j}$, then compute $\int_C \overline{F} \cdot d\overline{r}$ about a circle C in the xy plane with center at origin and radius 2, if C is traversed in the positive direction.

5×3

6. (a) Show that :

$$\frac{h}{1 + h^2} < \tan^{-1} h < h$$

when $h > 0$.

(*b*) Evaluate :

$$\nabla^2\left[\nabla \cdot \left(\frac{\vec{r}}{r^2}\right)\right]$$

(*c*) Discuss the convergence of the series :

$$\sum\left[\frac{n^2+1}{n^2+5}\right]x^n,\ x > 0.$$

7. (*a*) Find the maximum value of $x^m y^n z^p$ when $x + y + z = a$.

(*b*) Trace the curve $r = a \sin 3\theta$

(*c*) Find the area of the surface generated when the loop of the $9ay^2 = x(3a - x^2)$ revolved about x-axis.