This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 6470

Unique Paper Code : **235271** **D**

Name of the Paper : **Mathematics—I [MAHT-204]**

Name of the Course : **B.Sc. (Hons.) Electronics**

(Admissions of 2010 and onwards)

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are **7** questions in this question paper.

Question No. **1** is compulsory.

Attempt any *four* questions from Question No. **2** to Question No. **7**.

(*a*) State ratio test for a positive term series and hence show that the series : 5×3

$$\sum \frac{n}{2^n}$$

is convergent.

(*b*) Find the total differential coefficient of x^2y with respect to 'x' when x and y are connected by :

$$x^2 + xy + y^2 = 1.$$

(*c*) Find the perimeter of the curve :

$$r = a(\cos \theta + \sin \theta);\ 0 \leq \theta \leq \pi.$$

(*d*) Evaluate :

$$\int_0^1 \int_0^x \int_0^{x+y} (x + y + z)\, dz\, dy\, dx .$$

(*e*) If

$$\overline{A}(u) = u\hat{i} - u^2\hat{j} + (u - 1)\hat{k} \text{ and}$$

$$\overline{B}(u) = 2u^2\hat{i} + 6u\hat{k},$$

then evaluate :

$$\int_0^2 \overline{A} \times \overline{B}\, du .$$

2. (*a*) If $<a_n>$ is a sequence where $a_1 = 1$ and 3×5

$$a_{n+1} = \sqrt{2 + a_n} \quad \forall\ n \geq 1,$$

show that the sequence $<a_n>$ is monotonically increasing and bounded. What is $\lim_{n \to \infty} a_n$?

(*b*) Show that :

$$x - \frac{x^2}{2} < \log(1 + x) < x - \frac{x^2}{2(1 + x)} \quad \forall\ x > 0 .$$

(c) State Leibnitz test for the alternating series :

$$u_1 - u_2 + u_3 - u_4 + \dots\dots\dots \quad (u_n > 0, \ \forall n)$$

Show that the series :

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots\dots$$

is conditionally convergent.

3. (a) Expand : 3×5

$$f(x, y) = e^y \log (1 + x)$$

in powers of x and y upto third degree terms.

(b) Find the shortest distance from origin to the surface :

$$xyz^2 = 2.$$

(c) Trace the curve :

$$y = \frac{x^2 - 3x}{x - 1}.$$

4. (a) Calculate the area of the surface of revolution generated by revolving the cardioid :

$$x = 2\cos \theta - \cos 2\theta;$$

$$y = 2\sin \theta - \sin 2\theta;$$

about x-axis.

(*b*) Find the mass and centroid of the tetrahedron bounded by the coordinate planes and the plane :

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

(*c*) Using spherical polar coordinates, calculate : 3×5

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1 - x^2 - y^2 - z^2}}.$$

5. (*a*) Find the directional derivative of $\nabla.\bar{u}$ at the point (4, 4, 2) in the direction of the corresponding outer normal of the sphere :

$$x^2 + y^2 + z^2 = 36$$

where $\bar{u} = x^4\hat{i} + y^4\hat{j} + z^4\hat{k}$.

(*b*) Verify Stokes theorem for :

$$\bar{A} = y^2\hat{i} + xy\hat{j} - xz\hat{k}$$

over S, where S is the hemisphere :

$$x^2 + y^2 + z^2 = a^2;\ z \geq 0.$$

(*c*) If $f(r)$ is differentiable and

$$r = \sqrt{x^2 + y^2 + z^2},$$

show that $f(r)\ \bar{r}$ is irrotational. 3×5

6. (*a*) If

$$0 \leq a < b < \left(\frac{\pi}{2}\right),$$

show that :

$$0 < \cos(a) - \cos(b) < b - a.$$

(*b*) Find the surface area of the plane :

$$x + 2y + 2z = 12$$

cut-off by $x = 0$, $y = 0$ and $x^2 + y^2 = 16$ in the first octant.

(*c*) Discuss the convergence of the series

$$\sum \frac{n}{\left(n^2+1\right)^2}.$$

using Cauchy's integral test. 3×5

7. (*a*) If

$$\sin^2 u = \frac{x^{\frac{1}{3}} + y^{\frac{1}{3}}}{x^{\frac{1}{2}} + y^{\frac{1}{2}}},$$

then show that :

$$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = \frac{\tan u}{12}\left[\frac{13}{12} + \frac{\tan^2 u}{12}\right].$$

(*b*) Trace the curve :

$$r = a \cos 3\theta.$$

(*c*) Evaluate :

$$\iint_R (x+y)^2 \, dxdy,$$

where R is the region bounded by the parallelogram : 3×5

$$x + y = 0;$$

$$x + y = 2;$$

$$3x - 2y = 0;$$

$$3x - 2y = 3.$$