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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1085**

Unique Paper Code : **235271** **E**

Name of the Paper : **Mathematics-I (MAHT-204)**

Name of the Course : **B.Sc. (H) Electronics-I**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are **8** questions. Attempt any *two* parts from each question.

1\. (*a*) Discuss the convergence of the sequence $<a_n>$, where :

$$a_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \ldots\ldots\ldots + \frac{1}{(n+n)^2}.$$ 4½

(*b*) Test for convergence the following series :

(*i*) $\sum \left[\frac{\sqrt{n+1} - \sqrt{n-1}}{n}\right]$ 2

(*ii*) $\sum (n!)x^n$, where $x > 0$. 2½

(*c*) State the Cauchy's Integral test for the convergence of an infinite series. Use it to prove the convergence of the series $\sum \frac{1}{n^2}$. 4½

2\. (*a*) Show that :

$$\frac{h}{1+h^2} < \tan^{-1} h < h,$$

where $h > 0$. 4½

P.T.O.

(*b*) State Maclaurin's theorem with Lagrange's form of remainder for $f(x) = \cos x$. 4½

(*c*) State Lagrange's Mean value theorem. Use it to calculate approximately the root of the equation $x^{14} - 12x + 7 = 0$ near 2. 4½

3. (*a*) Define :

$$f(x) = \begin{cases} \dfrac{x}{\sqrt{x^2 + y^2}} & \text{if } (x, y) \neq (0, 0) \\ 3 & \text{if } (x, y) = (0, 0) \end{cases}$$

Show that the function 'f' is discontinuous at the origin. 4½

(*b*) If

$$u = \log\left(\frac{x^2 + y^2}{x + y}\right),$$

prove that $xu_x + yu_y = 1$. 4½

(*c*) Define whether the functions u and v defined as following are functionally dependent or not. Find a functional relation between them in case they are functionally dependent :

$$u = \frac{x}{y} \text{ and } v = \frac{x + y}{x - y}.$$ 4½

4. (*a*) Find the shortest distance from the origin to the surface $xyz^2 = 2$. 4½

(*b*) Trace the curve $y^2 \cdot (a - x) = x^3;\ a > 0$. 4½

(*c*) Evaluate :

$$\int_{-2}^{1} \int_{x^2 + 4x}^{3x + 2} dy dx.$$ 4½

5. (*a*) Determine the length of one loop of the curve $6ay^2 = x \cdot (x - 2a)^2$. 4½

(*b*) Determine the volume of solid generated by revolving the plane area bounded by $y^2 = 4x$ and $x = 4$ about the line $x = 4$. 4½

(c) Show that the surface area of the surface generated when the loop of the curve $9ay^2 = x \cdot (3a - x)^2$ revolves about the x-axis is $3\pi a^2$. 4½

6. (a) Show that the curve $x = a \cdot (\theta - \sin\theta)$, $y = a \cdot (1 - \cos\theta)$ is divided in the ratio $1 : 3$ at $\theta = 2\pi/3$. 5

(b) Calculate the volume of the solid of revolution generated by revolving the hypocycloid :

$$\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1,$$

about x-axis. 5

(c) Evaluate :

$$\iiint_V xyz\left(x^2 + y^2 + z^2\right)^{\frac{n}{2}} dxdydz$$

taken through the positive octant of the sphere $x^2 + y^2 + z^2 = b^2$ provided $n + 5 > 0$. 5

7. (a) Find the directional derivative of $\nabla \cdot \vec{U}$, at the point (4, 4, 2) in the direction of the corresponding outward drawn normal to the sphere $x^2 + y^2 + z^2 = 36$, where :

$$\vec{U} = x^4\hat{i} + y^4\hat{j} + z^4\hat{k}.$$ 5

(b) Prove that :

$$\vec{F} = \left(y^2 - z^2 + 3yz - 2x\right)\hat{i} + \left(3xz + 2xy\right)\hat{j} + \left(3xy - 2xz + 2z\right)\hat{k}$$

is both solenoidal and irrotational. 5

(c) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that $\nabla^2(\log r) = \frac{1}{r^2}$. 5

8. (*a*) Verify Green's theorem in the plane for :

$$\oint_C \left(3x^2 - 8y^2\right)dx + \left(4y - 6xy\right)dy.$$

where C is the boundary of the region defined by $y = \sqrt{x}$ and $y = x^2$. 5

(*b*) Apply Stokes' Theorem or otherwise evaluate :

$$\int_C \left(ydx + zdy + xdz\right)$$

where C is the curve of intersection $x^2 + y^2 + z^2 = a^2$ and $x + z = a$. 5

(*c*) Use divergence theorem or otherwise evaluate :

$$\iint_S \left(xdydz + ydzdx + zdxdy\right)$$

over the surface of the sphere of radius 'a'. 5