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Your Roll No. .....

1404

B.Sc. (Hons.) / II A

ELECTRONIC SCIENCE – Paper 2.6 (XIII)

(Mathematical Physics – III)

Time : 3 Hours **Maximum Marks : 38**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **five** questions in all, including Question No. **1**, which is compulsory.

1. Do any **five** parts : 2×5

   (a) Given a complex number z, interpret geometrically $ze^{i\alpha}$, where α is the real number.

   (b) Identify the singularity of $f(z) = \dfrac{1 - e^z}{1 + e^z}$ at $z = \infty$.

   (c) Graph the region represented by
   $2 < |z - 4 - 5i| < 4$
   on the Argand's diagram.

(d) Evaluate : $\int_0^{\infty} 3^{-4z^2}\, dz$

(e) Find all the singular points of

$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy = 0$ and determine their nature.

(f) Show that the general solution of one dimensional wave equation can be represented by

$$y(x, t) = \frac{1}{2}\left[f_1(x + ct) + f_2(x - ct)\right]$$

where the symbols have usual meaning.

2. (a) Express the following in polar form and find the principal value of the function given by $\left(\frac{1-i}{1+i}\right)^{1/3}$. 2

(b) If $| z_1 + z_2 | = | z_1 - z_2 |$, find the value of $\arg z_1 - \arg z_2$. 2

(c) Expand $\ln\left(\frac{1+z}{1-z}\right)$ in a Taylor's series around the origin. 3

3. (a) Obtain the necessary and sufficient conditions for a function f(z) to be analytic. 3

(b) Discuss the differentiability and analyticity of $f(z) = x^n + iy^n$, n being a +ve integer. 2

(c) Evaluate : $\oint_C (1 + z)\, e^{\left(-\frac{1}{z}\right)} \sin\left(\frac{1}{z}\right) dz$ 2

Where C is a unit circle around the origin.

4. Using the calculus of residues, evaluate any **two :** $3\frac{1}{2} \times 2$

(a) $\int_0^{\pi} \frac{1 + 2\cos\theta}{5 + 4\cos\theta}\, d\theta$

(b) $\int_0^{\infty} \frac{x \sin 2x}{x^2 + a^2}\, dx$

(c) $\int_0^{\infty} \frac{\sin x}{x}\, dx$

5. (a) Show that : 2

$$\beta(m, n) = \frac{\lceil m \ \lceil n}{\lceil (m + n)}$$

Where $m > 0$, $n > 0$.

(b) Solve the differential equation 5

$4x^2 y'' + 4xy' + (x^2 - 1)y = 0$

by Frobenius method.

6. (a) Prove that $\left|\left(\frac{1}{2}\right)\right| = \sqrt{\pi}$ 2

(b) $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}\, dx = \beta(m, n)$ 1

(c) Prove that : $\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx = 0,\ \alpha \neq \beta$ 4

Where α and β are the zeros of $J_n(x)$.

7. (a) Using Rodrigue's formula for Legendre's Polynomial $P_n(x) = \frac{1}{2^n\, n\,!}\, \frac{d^n}{dx^n}\left(x^2 - 1\right)^n$ 3

Show that $\int_{-1}^1 \left[P_n(x)\right]^2 dx = \frac{2}{2^{n+1}}$

(b) Prove : 2

${}^n p_n(x) = (2n-1)x\, p_{n-1}(x) - (n-1)\, p_{n-2}(x)$

(c) Compute $J_{1/2}(x)$. 2

8. Set up the two dimensional wave equations for a vibrating circular membrane specifying the relevant boundary and initial conditions. Obtain its solution. 7
