[This question paper contains 4 printed pages.]

1013 *Your Roll No.* ....

B.Sc. (Hons.) / II **C**

ELECTRONIC SCIENCE – Paper 2.6 (XIII)

(Mathematical Physics – III)

Time : 3 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **five** *questions in all, including Question No.* **1** *which is compulsory.*

1. (a) Compute the principal value of $f(z) = i^i$.

(b) Determine the total number of branches of

$$f(z) = z^{\frac{1}{2}} + z^{\frac{1}{3}}$$

(c) Evaluate : $\oint_C e^{\frac{1}{z}} \sin\left(\frac{1}{z}\right) dz$

C being a unit circle.

P.T.O.

(d) Prove that $P_n''(1) = \frac{1}{8}n\left(n^2 - 1\right)\left(n + 2\right)$.

(e) Verify that $P(n) = \int_0^1 \left(\ln \frac{1}{x}\right)^{n-1} dx,\ n > 0$. (2×5)

2. (a) Establish the necessary and sufficient conditions for f(z), a function of complex variables to be analytic. (5)

(b) Prove that

$J_{-n}(x) = (-1)^n J_n(x)$, for integral 'n' values. (2)

3. (a) State and prove Laurent theorem. (4)

(b) Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in Laurent series valid for

(i) $|z| > 2$

(ii) $0 < |z - 2| < 1$ (3)

4. (a) If F(z) is analytic inside and on a simple closed curve C except for a pole of order 'n' at z = a inside C prove that

$$\frac{1}{2\pi i}\oint_C F(z)dz = \underset{z \to a}{\text{Lt}} \frac{1}{(n-1)!} \frac{d^{n-1}}{dz^{n-1}} \left\{(z-a)^n F(z)\right\}$$

(4)

(b) Evaluate : $\oint_C \frac{e^z\, dz}{(z^2+\pi^2)^2}$, C being a circle $|z| = 3.5$.

(5)

5. Evaluate the following integrals :

(a) $\int_0^{\pi} \frac{d\theta}{(2+\cos\theta)^2}$ (3)

(b) $\int_0^{\infty} \frac{\sin 2x}{x}\, dx$ (4)

6. (a) Solve the following differential equation by Frobenius method :

$(1-x^2)y'' + 2xy' + y = 0$ (5)

(b) Evaluate : $\int_{-1}^{1} (1-x)^a (1-x)^b\, dx$,

$a, b > 0$ (2)

7. (a) Prove that $L_n(x)$ satisfies the orthogonality relation :

P.T.O.

$$\int_0^{\infty} e^{-x} L_m(x) L_n(x)\, dx = \delta_{mn} \qquad (3)$$

(b) Find the fundamental frequency of transverse vibrations of a square membrane of unit dimensions. (4)