[This question paper contains 5 printed pages.]

6005 Your Roll No. ....

B.Sc. (Hons.) Electronics / III Sem. **B**

Paper – MAHT – 305

Mathematics – II

(Admissions of 2010 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are two Sections. All parts of Question No. **1** *in Section–***I** *is compulsory. Attempt any* **four** *questions from Section* **II.** *Marks are as indicated.*

SECTION – I

1. (a) Let $v_1 = (1, -1, 0)$, $v_2 = (0, 1, -1)$ and $v_3 = (0, 0, 1)$ be three elements of $\mathbb{R}^3$. Show that the set of vectors $\{v_1, v_2, v_3\}$ is linearly independent. (3)

(b) Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$ (3)

(c) Find the nature, index and signature of quadratic form.

$$2x_1x_2 + 2x_1x_3 + 2x_2x_3.$$ (3)

P.T.O.

(d) Show that $f(z) = \operatorname{Re} z = x$ is continuous but not differentiable. (3)

(e) Classify each of the following differential equation by its kind, order and degree.

(i) $\frac{\partial u}{\partial t} = K\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$

(ii) $\left(\frac{dr}{ds}\right)^3 = \sqrt{\frac{d^2 r}{ds^2} + 1}$ (3)

SECTION – II

2. (a) Find the dimension of the sub-space of $\mathbb{R}^4$ spanned by the set {(1 0 0 0), (0 1 0 0), (1 2 0 1), (0 0 0 1)}. Also find 0 basis of the subspace. (5)

(b) Determine the values of 'a' and 'b' for which the system.

$$x + 2y + 3z = 6$$

$$x + 3y + 5z = 9$$

$$2x + 5y + az = b$$

has (i) no solution (ii) unique solution (iii) infinite number of solutions. Find the solutions in case (ii). (5)

(c) Solve the following system of equations by Gaussian elimination method.

$$2x_1 + x_2 + 3x_3 = 1$$
$$4x_1 + 4x_2 + 7x_3 = 1$$
$$2x_1 + 5x_2 + 9x_3 = 3 \quad (5)$$

3. (a) Verify the Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

and hence find the inverse of A. (5)

(b) Show that any square matrix 'A' can be written as the sum of a symmetric matrix 'B' and skew-symmetric matrix 'C'. (5)

(c) Show that $A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$

is Skew-Hermitian and also unitary. Find the eigen values. (5)

4. (a) Find the analytic function

$f(z) = u + iv$, where

$u = e^x (x \cos y - y \sin y) +$

$2 \sin x . \sinh y + x^3 - 3xy^2 + y$. (5)

(b) Evaluate, using Cauchy's integral formula :

$\int_C \frac{\cos \pi z}{z^2 - 1} dz$, around a rectangle with vertices $2 \pm i$, $-2 \pm i$ (5)

(c) Determine the residues at all its poles of the function

$$f(z) = \frac{z^2}{(z-1)^2 (z+2)}$$

Hence evaluate $\int_C f(z)\, dz$, where C is the Circle $|z| = 2.5$. (5)

5. (a) Solve :–

$$\left(1 + 2e^{\frac{x}{y}}\right) + 2e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) \cdot \frac{dy}{dx} = 0 \qquad (5)$$

(b) Solve :–

$$(1 + y^2)dx + (x - \tan^{-1} y)dy = 0 \qquad (5)$$

(c) Find particular member of orthogonal trajectories of $x^2 + cy^2 = 1$ passing through the point (2, 1). (5)

6. (a) Solve $AX = B$ by LU – decomposition using Gaussian elimination where

$$A = \begin{pmatrix} 2 & 4 & -6 \\ 1 & 5 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

and $B^T = (-4, 10, 5)$. (5)

(b) Find the orthogonal transformation which transforms the quadratic form

$$x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$$

to canonical form. (5)

(c) Find the Laurent series expansion of

$$f(z) = \frac{e^z}{z(1-z)}$$

about $z = 1$. Find the region of convergence of the expansion. (5)

7. (a) Evaluate

$$I = \int_0^{\pi} \frac{\cos^2 3\theta \, d\theta}{(5 - 4\cos 2\theta)}$$ (5)

(b) Solve :

$$(x - 2y + 1)dx + (4x - 3y - 6)dy = 0$$ (5)

(c) Solve :

$$(y + x)dy = (y - x)dx$$ (5)